

The Use of Simple Geometry and the Local Unit of Measurement in the Design of Italian Stringed Keyboard Instruments: An Aid to Attribution and to Organological Analysis

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Source: The Galpin Society Journal, Vol. 52 (Apr., 1999), pp. 108-171

Published by: Galpin Society

Stable URL: http://www.jstor.org/stable/842520

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### **GRANT O'BRIEN**

# The use of simple geometry and the local unit of measurement in the design of Italian stringed keyboard instruments: an aid to attribution and to organological analysis

This paper is dedicated to the memory of the late John Barnes who has been, and still is, one of the most important influences on my life and work

More than half of the 750 or so Italian harpsichords, virginals and spinets which survive from the historical period are unsigned. Of the signed instruments a significant number either bear false signatures or are falsely attributed, and therefore neither their maker nor the centre in which they were built is known. The lack of biographical information about a number of the makers of instruments with signatures that appear to be authentic means that we do not know where they lived and worked. This situation is clearly detrimental to an understanding of the stringed keyboard instrument building tradition in the Italian peninsula.

Because the Italian peninsula was divided politically into separate city and church states during the historical period of stringed keyboard instrument making, and because these regions remained to a certain extent individual and distinct, and often isolated from one another, the building of Italian harpsichords and virginals followed somewhat different paths from one locality to another. This means that, although Italian harpsichords and virginals are superficially similar in a number of ways, there are many features of their construction, stringing, disposition and acoustical and musical properties that are different from one region to the next. Understanding exactly the extent and nature of these differences will clearly not be possible until the surviving instruments, including the large number of anonymous instruments, are grouped according to the geographical region in which they were built.

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Grouping the instruments in this way, the musical resources of the extant instruments can be related to the music and musical traditions of the regions in which they were built. This is of course extremely important to the history of performance practice since it makes clear what the musical resources of the keyboard instruments were in each of the different regions and periods. This helps us to understand what is and is not possible musically based on the surviving instruments. The study of these regional differences within Italy is of great interest not only to scholars studying the history of early stringed keyboard instruments and the music performed on them, but also to modern instrument makers who are copying old instruments for use in making music in the present early music revival, and to the non-scholar musicians playing early Italian music. The most important hurdle to be overcome in this study is to identify the area in which the anonymous unsigned instruments were made.

Another aspect of the regional variations in the history of stringed keyboard instrument making in Italy concerns the modifications which these instruments underwent during the historical period in order to bring them up to date. Sometimes the maker responsible for the modifications is known, but usually he is not. However, the modifications are just as important to the history of performance practice and to the changing styles, pitch levels, musical resources, etc. as are the unaltered instruments. There has hitherto been no method of establishing the area in which these modifications were carried out.

### THE LOCAL UNITS OF MEASUREMENT

During the whole of the historical period of harpsichord and virginal building up to the beginning of the nineteenth century, virtually every large city and major centre in Italy used a differently-sized unit of measurement. Again this was a result of the political division of the peninsula into separate church and city states each with its own standards of length, weight, fluid measure and currency. In most of the centres the basic, larger unit of measurement was usually either the *piede*, *palmo* or *braccio* (the *passo*, *passetto* and *raso* were also used¹) and these were divided into the *oncia* or sometimes the *soldo* or the *pollice*. Only in the period after the Napoleonic invasions of the Italian peninsula, and therefore after the historical period of harpsichord and virginal building, did the metre replace the various local units of measurement. Therefore if the unit of measurement used in the design and construction of an instrument can be

<sup>&</sup>lt;sup>1</sup> A number of other units of measurement which are too large to be involved in musical instrument making like the *canna*, *cannella*, *tesa*, *trabucco*, *pertica*, *cavezzo*, *corda* and *catena* were also in use.

determined, this can be used in turn to establish the centre of its origin. This is a fundamental procedure, basic to the process of establishing the region in which an instrument originated, and can be a great help in establishing the maker of an otherwise anonymous instrument.

The ability to establish the unit of measurement used to construct a radically-modified instrument is also basic to any reconstruction of its original state. The method described below has been applied to the Russell Collection Stefano Bolcioni 1627 three-manual harpsichord² which has undergone a drastic alteration to its original case dimensions, disposition, string scalings and pitch from its original single-manual state. This aspect of the use of the unit of measurement as a powerful tool in the analysis of the alterations to this instrument will be elaborated in a further article to be published next year in this Journal.

Appendix 2 at the end of this article gives values of the local units of measurement in the centres throughout Italy where harpsichord and virginal builders are known to have worked. These are arranged both according to location and also according to the size of the *oncia*, *soldo* and *pollice*, and some of the measurements from these tables will be used in the study of some of the instruments in the subsequent discussion. Clearly the lengths of the various units of measurement from these tables can also be used in the investigation of further instruments by anyone wishing to analyse them in a manner similar to that described below.

# THE BASEBOARD LAYOUT AND DESIGN OF ITALIAN POLYGONAL VIRGINALS

It is quite clear that any maker of instruments — or any other object for that matter — would have worked on a day-to-day basis using convenient numbers and uncomplicated fractions of his local unit of measurement. For the plain reason that whole numbers or simple fractions are easy to remember, an artisan would work in convenient units of measurement when he is designing and executing the object he is making. Because most of the measurements used are, to a certain extent at least, arbitrary there is no need to invoke complicated numbers in their design. This is clear to anyone who has lived or worked anywhere in much of the English-speaking world where the inch, a twelfth part of a foot, was until recently still being used. Most of the measurements used by handworkers, artisans, artists, architects, designers and all of those involved in industry and commerce were based on simple numbers of inches, feet and yards, or on their simple subdivision.

<sup>&</sup>lt;sup>2</sup> See Sidney Newman and Peter Williams, The Russell Collection and other Early Keyboard Instruments in Saint Cecilia's Hall, Edinburgh, (Edinburgh, 1968) Catalogue Number 4, frontispiece, viii, pp.8-9. The new Russell Collection inventory number of this harpsichord is HT1-SB1627.4.

Only where it is really necessary and where dictated by some rule or theoretical concept would an instrument builder use a complicated or irrational division of the local unit of measurement. This has been shown clearly from the ground-breaking study made of many types of musical instruments by Herbert Heyde.<sup>3</sup> But what Heyde, and more recently Hubert Henkel,4 have failed to note is that the makers of Italian stringed keyboard instruments, at least, designed their instruments beginning with the baseboard and then worked literally from there upwards. The instrument case measurements used by Hevde and Henkel to suggest theories of numerology in instrument building have been taken (incorrectly in my view) for Italian instruments from the outside case dimensions including the case-side thicknesses, although not including the added measurements of the upper or lower mouldings. For instruments built in the Italian tradition where the case sides are applied to the outside edges of the baseboard, the maker clearly began both the design and the actual construction with the baseboard. It is therefore the measurements of the baseboard that reflect this. The measurements of stringed keyboard instruments which have been used by Heyde and Henkel, however, take no account of the dimensions of the baseboard before the case-side planks were added, but are instead based on the dimensions of the case after the sides are added, and after the top moulding is added to the top edge of the case sides.

In contrast, the work that I have done recently in this field and illustrated below shows that the maker began his design by drawing out the baseboard using dimensions which were simple integers or fractions of the local unit of measurement, and the case sides that he then applied to the outer edges of the baseboard were cut to a height also equal to a simple number of units (or units plus simple fractions) of the local measurement unit. The combination of the fact that the case sides were hand thicknessed and therefore not all of exactly the same thickness (not even from one end of the board to the other) and the irregular geometry of both polygonal virginals and harpsichords, meant that the final outside dimensions of the instrument were totally unrelated to the local unit of measurement used by the maker. Therefore a maker starting with two identically-dimensioned baseboards constructed according to his design could end up with slightly differently-sized cases after the sides were added to the two identical baseboards. Similarly it is the height of the case without the top cap moulding that the maker would measure in his local unit of

<sup>&</sup>lt;sup>3</sup> See: Herbert Heyde, Musikinstrumentenbau, 15.-19. Jahrhundert. Kunst Handwerk Entwurf, (VEB Deutscher Verlag für Musik, Leipzig, 1986).

<sup>&</sup>lt;sup>4</sup> See: Hubert Henkel, Besaitete Tasteninstrumente. Fachbuchreihe das Musikinstrument, Vol. 57 (Verlag Erwin Bochinsky, Frankfurt-am-Main, 1994).

measurement.<sup>5</sup> He would mark out a number of planks all of the same width in convenient units and then cut and apply these to the outside edges of the baseboard. Experience has shown that even here, the casewall heights are often slightly less than expected in places where the top of the case has been planed down to equalise the level of the top edges at the corners when these did not match exactly after the case sides were assembled. It is therefore the maximum case-wall height that corresponds to the makers design and not the average case-wall height. Similarly the position of the soundboard was located by choosing a simple distance for the top of the soundboard liner relative to the top or the bottom edge of the case sides. The bottom of the soundboard was therefore not positioned relative to the upper surface of the baseboard, and similarly the top of the soundboard (which was usually of slightly variable thickness for acoustical reasons) was similarly also unrelated in simple units of the local measurement to the position of the top or bottom of the case. Clearly, which measurements were chosen by a maker in simple units would depend on his method of working and especially on the order in which the various operations necessary to construct the instrument were carried out.

The problem faced by an investigator is to find the unit of measurement used to design and construct any given instrument. An instrument has many different measurements and it is not at all obvious from looking at these expressed in millimetres what the local unit used to arrive at them was. Even knowing that the baseboard was constructed using simple units of the local measurement is not, in itself, enough to divine the length of the unit used in its design. This is further complicated by the fact that, being hand made, none of the measurements of the baseboard or the rest of the case, keyboards, scalings, etc. is perfectly exact. Any method used to find the unit of measurement must therefore also be relatively insensitive to any inaccuracies resulting from the working methods of the maker.

I want to show here, first of all, how some of the basic principles used by Italian makers when setting out their design for the baseboards of both polygonal virginals and harpsichords were based on the local unit of measurement. The method used by these makers is based on the way in which they used a simple geometrical construction to arrive at the corner angles of polygonal virginals, and in a similar way to arrive at the tail angle of harpsichords. Working in reverse, a study of the measurement of the angle and of the orthogonal components of the sides of these corners enables a calculation of the unit of measurement.

<sup>&</sup>lt;sup>5</sup> In the North-European tradition where the case sides are much thicker than in Italian practice, the top moulding is often cut into the wood of the case side itself, and the case sides are usually (but not always) applied to the top of the baseboard. It is therefore the case height less the thickness of the baseboard that the maker would measure out using a simple number of local units.

Establishing the unit of measurement used in the design of the instrument can then be used to determine the centre in which it was built.<sup>6</sup>

The importance of such a method to the determination of the maker of an anonymous instrument is obvious. The method clearly does not pinpoint precisely who the maker was, but it does reduce the number of possible makers from the vast breadth of Italian harpsichord, virginal and spinet builders active across the whole of the peninsula to those working in one area or centre. It thus accelerates greatly the process of an eventual attribution of the instrument. Once the area in which it was made has been determined, it suffices then to compare the anonymous instrument in question with other similar instruments by known builders from the same city or region.

I want to illustrate the method that I have developed to arrive at the unit of measurement for both harpsichords and virginals. First of all I will examine the design of a polygonal virginal by Franciscus Patavinus, and I will then illustrate a simple application of the procedure that I have developed to determine where the makers Marcus Siculus and Ignazio Mucciardi, about both of whom we have no biographical information, worked. I then want to use the method to establish the unit of measurement used by Stefano Bolcioni working in Florence. This will be done beginning with the measurements of the baseboards of a virginal and a harpsichord by him, and then the length of the unit of measurement will be compared with the known value of the unit of measurement used in Florence. Having established the unit of measurement used by Bolcioni I then want, in a subsequent paper in next year's volume of this Journal, to show how a knowledge of this unit can be crucial to the reconstruction of the original state of the Edinburgh Russell Collection Bolcioni harpsichord mentioned above. Other methods of determining the local unit of measurement used in an instrument are also then discussed.

# A VIRGINAL BY FRANCISCUS PATAVINUS DATED 1552 IN THE MUSEO CORRER, VENICE

As explained above, in Venice and throughout the rest of the Italian peninsula, the baseboard dimensions without the case sides were chosen

<sup>&</sup>lt;sup>6</sup> Denzil Wraight, in his otherwise splendid work on the identification of Italian keyboard instruments, rejects the evidence provided by the local unit of measurement (see: Denzil Wraight, 'The identification and authentication of Italian string keyboard instruments', *The Historical Harpsichord. Volume Three*, general editor Howard Schott, (Pendragon Press, Stuyvesant, NY, 1992) pp.66-76). Unfortunately he seems to discount the whole process of using the local unit of measurement as a method for determining the origin of an instrument on the basis of a quoted example of the confusion that has arisen because of the fact that the Frankfurt and Vicenza inches are fortuitously in the ratio of 3 to 4.

in simple units or fractions of the inch or oncia<sup>7</sup> (plural once) that the maker was using. Since the oncia was normally divided into twelve equal parts each called a line or linea (plural linee) it is to be expected that fractions involving twelfths, sixths, thirds, quarters and halves of the oncia would be involved in the design and execution of the instruments.<sup>8</sup> The Venetian foot or piede<sup>9</sup> (plural piedi) had a length close to 347.76mm,<sup>10</sup> and this was divided into 12 giving an inch or oncia of 28.98mm.

The Museo Correr on the Piazza San Marco in Venice holds a fine Italian virginal signed: ' ~ FRANCISCI PATAVINI DICTI HONGARO MDLII ~ '.<sup>11</sup> The namebatten and the signature are definitely not original to the instrument. The signature is written on a piece of wood foreign to the rest of the instrument, and this wood appears to be fir or spruce stained brown to match the appearance of the cypress used elsewhere in the instrument. The fact that the nameboard and signature are not original to the instrument does not, however, mean

<sup>&</sup>lt;sup>7</sup> The words inch, ounce and *oncia* all derive from the Latin word *uncia* meaning 'a twelfth part'. Therefore an inch is a twelfth part of a foot and a troy ounce is a twelfth part of a troy pound. However there are a number of cases, such as the normal English pound weight, where the division was into 16 ounces and not into 12. In Rome the *piede* was divided into 16 *once* and existed alongside the Roman *palmo* which had 12 *once* (hence 1 *piede* = 1½ *palmi*). Other divisions are also possible as in Florence, for example. Here the *braccio* was divided into 2 *palmi* each of length 10 *soldi* (*soldo* in the singular). Therefore the *braccio* had a length of 20 *soldi*.

<sup>&</sup>lt;sup>8</sup> This is not always true, however, and sometimes the *piede* and *palmo* were divided into 10 units, and sometimes the subdivisions were also in 10 units. In Rome, for example, the *oncia*, a twelfth part of the *palmo*, was divided into 5 *minuti* and 10 *decimi*.

<sup>&</sup>lt;sup>9</sup> See my article, 'Marco Jadra. A Venetian harpsichord and virginal builder?', Gedenkschrift für Kurt Wittmayer, to be published in 1999 and edited by Silke Berdux, for a discussion of a number of instruments built using the Venetian foot or piede.

<sup>&</sup>lt;sup>10</sup> See: Colonel Cotty, Aide-mémoire a l'usage des officiers d'artillerie de France, 2 (Magime, Anselin & Pochard, Paris, 5/1819) p.899 (here 1 Venetian piede = 347.7588mm so that the oncia = 28.9799mm). The Venetian piede is among the best-documented units of measurement and various sources give values from 347.398mm to 347.759mm (see Appendix 2 at the end of this paper).

<sup>&</sup>lt;sup>11</sup> I have examined this instrument in some detail during the course of a study project organised by the Museo Correr and by Il Laboratorio of Milan and indeed it was during the study of this virginal for the Museo Correr that I discovered the simple geometry used to design the baseboard. An unpublished report entitled *Spinetta poligonale Franciscus Patavinus*, 1552 written by me for this project is held by the Museo Correr in Venice

that its maker is not Francesco Patavinus.<sup>12</sup> Indeed the mouldings on the instrument are not even the same as those of the two other extant instruments thought to be by Franciscus Patavinus.<sup>13</sup> However here, as with other makers I have studied where there is a lack of correspondence of the mouldings, I do not see any reason for doubting that any of these instruments are by Patavinus.<sup>14</sup> Although he seems consistently to have signed himself 'FRANCISCVS PATAVINVS DICTI [H]ONGARO' so that he appears both to have had Hungarian roots and to have come from Padova, he is almost certainly to be identified with the 'Francesco dalli arpicordi' and the 'Francesco dai manicordi' who appears in the Venetian archives<sup>15</sup> and who lived and worked there.

Figure 1 shows a schematic representation of the case mouldings, the keywell scrolls, and the bridge section at the position of the  $c^2$  string of the 1552 polygonal virginal by Franciscus Patavinus in the Museo Correr, Venice. The application of the case sides to the outer edge of the baseboard, and the additional height of the case sides resulting from the

<sup>&</sup>lt;sup>12</sup> The new namebatten may have been made for the instrument when the old, original namebatten went missing or was damaged for whatever reason. In fact this seems highly likely since it is improbable that the appellation 'DICTI HONGARO' would have been used by someone attributing the instrument to Franciscus unless he was sure of the original form of the signature.

<sup>13</sup> Donald H Boalch, Makers of the Harpsichord and Clavichord, 1440-1840, (Third edition, edited by Charles Mould, Clarendon Press, Oxford, 1995) pp.319-320 lists altogether 4 instruments by Patavinus. The second of these is listed only in the catalogues by Franciolini (see: Edwin M. Ripin, 'The instrument catalogues of Leopoldo Franciolini', Music Indexes and Bibliographies, Vol. 9 (New Jersey, 1974) 3A-14, p.14) as an instrument signed 'IONNES[sic] PATAVINI[sic] DECTI[sic] HONGARI[sic] MDXXXX'. In addition there is a polygonal virginal in the Brussels Museum of Musical Instruments (No.272) listed in Boalch/3 under Antonius (p.222) with a signature 'ANTONI PATAVINI OPVS MDXXXXX[sic]' on a namebatten that does not belong to the instrument.

<sup>&</sup>lt;sup>14</sup> Besides numerous similar construction methods used, the bass ends of the boxslide registers of both of the virginals have the inscription 'bafi' = bassi written on one side, an indication to the maker while he was assembling the instrument which end of the boxslide was for the bass and which for the treble. The Florentine makers Francesco Poggio and Stefano Bolcioni also both use the word 'bassi' on the bass end of their virginal registers to indicate its orientation during the construction of the instrument. But I know of no other maker who used the form 'bafi' with a long  $\int =$  'ss', and no Venetian maker at all who left this indication on the bass end of the boxslide register.

<sup>&</sup>lt;sup>15</sup> See: Stefano Toffolo, Antichi Strumenti Veneziani. 1500-1800: Quattro secoli di liuteria e cembalaria, (Arsenale Editrice, Venice, 1987) pp. 161-2. The Italian word 'arpicordo' seems to have been used for what we now define as a virginal, or in modern Italian a 'spinetta' or, more properly, a 'spinetta traversa'. A 'manicordo' was the word used for clavichord.

extra depth added by the top cap moulding and ivory studs are clearly indicated here.

Unfortunately the usual catalogue measurements of Italian and Venetian virginals (Table 1 and Figure 2) are taken of the outer case sides and, to my knowledge, never of the baseboard on its own. Hence the normal catalogue measurements do not normally enable one to make any sort of an analysis of the size of the baseboard from which the maker began the design and construction of the instrument. It is therefore necessary to measure the baseboard without the case sides and then to analyse these measurements.

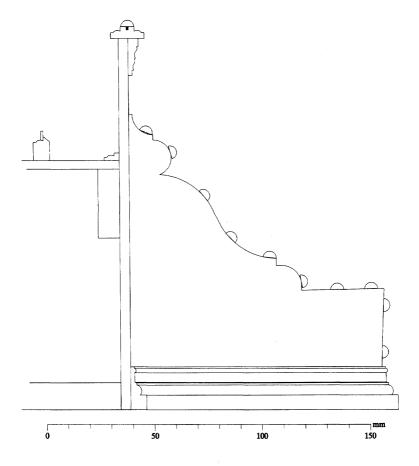


Fig. 1. Schematic representation of the case mouldings, the keywell scrolls, and the bridge section at the position of the c<sup>2</sup> bridge pin. Polygonal virginal by Franciscus Patavinus, 1552. Museo Correr, San Marco, Venice

Table 1

Outside dimensions in mm including the case sides, but not the outer mouldings.

Polygonal virginal by Franciscus Patavinus, 1552.

Museo Correr, San Marco, Venice.

	Dimension	Height*	Thickness	Wood
Front:	1641	172-4	5.4-6.4	cypress
Case left of the keywell:	344	173½-4	6.4	cypress
Angled left side:	192	173	4.7	cypress
Angled left back:	864	172	5.0	cypress
Back:	313	173	5.1	cypress
Angled right side:	571	172-3	5.2	cypress
Case right of the keywell:	569	172-4	5.4	cypress
Outside of the keywell:	728	-		-
Total width:	490	-		_
Keywell scrolls:	project 116	136	11	cypress
Baseboard:	Italian style		12.4-12.8	fir**
Angle at the left-front corner:			72°	
Angle	at the right-fr	ont corner:	41°	

<sup>\*</sup> These heights do not include the top cap moulding which adds a further 5mm to each measurement.

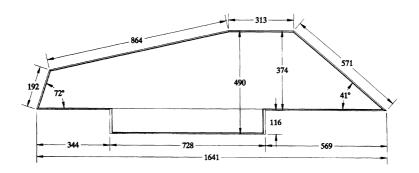


Fig. 2. Outer dimensions in mm including the case sides, but not the outer mouldings. Polygonal virginal by Franciscus Patavinus, 1552.

Museo Correr, San Marco, Venice

<sup>\*\*</sup> As there are no pitch pockets in this large piece of wood, it is almost certainly of fir and not of spruce.

Table 2 and Figure 3 show the baseboard measurements in millimetres of the 1552 Franciscus polygonal virginal without the case sides. A number of these are given in Table 2 in their nominal measurement in Venetian *once*. Many of these show a close agreement between the measured length in millimetres and a simple nominal number of Venetian *once*, and strongly suggest that this was the unit of measurement used in the design of the baseboard of this instrument. However the measurements of the sloping edges at the left- and right-hand sides of the case do not give measurements which can be expressed in whole numbers or simple divisions of the Venetian *oncia*. This suggests that the measurements of these sloping edges are not those that were used by the maker in the design of the instrument. The angles at the extreme ends of the virginal are also not simple numbers like 30°, 60° or 45°, or even simple angles based on multiples of 5° or 10°. These two facts must therefore somehow be related.

To understand this relationship and how the front corner angles were constructed it is necessary to examine their geometry. The tangent<sup>16</sup> of the angle at the left-hand corner, for example, is:

$$\tan 72^{\circ} = 3.07 \cong 3 = \frac{6}{2}$$

This suggests that the sloping surface at the left-hand side of the instrument was made up by drawing the hypotenuse of a triangle with orthogonal sides that are in the ratio of 3 once:1 oncia, 6 once:2 once, 9 once:3 once, etc. The actual measurement of the sloping side of just over 6 once immediately suggests that the two orthogonal sides of this triangle were designed by Patavinus to be 6 once and 2 once. Similarly at the right-hand corner the tangent of the angle there gives

$$\tan 41^\circ = 0.869 = \frac{12.5}{14.4} \cong \frac{12\frac{1}{2}}{14\frac{1}{2}}$$

and suggests that the angle formed at this corner resulted when Patavinus drew the hypotenuse of a right-angle triangle with sides 12½ (the width of the instrument) and 14½ once.

Figure 4 shows the dimensions in millimetres of the baseboard in directions perpendicular and parallel to the front of the instrument, and indicates the close agreement between the measurements at the front of the case with simple units of the Venetian *oncia*. Figure 5 shows the lengths of each of the sides of the baseboard of the Franciscus Patavinus virginal as it must have been designed by Franciscus, with the calculated angles at the front corners which would result from their construction using triangles with sides measured in simple numbers of Venetian *once*.

<sup>&</sup>lt;sup>16</sup> See Appendix 1 at the end of this paper for a brief and simple review of geometrical definitions.

The agreement between the measured values of both the lengths and of the front corner angles makes clear the design of the baseboard of this instrument by Franciscus in units of the Venetian *oncia*.

Other dimensions such as the maximum case height of 174mm (6.004 once) also give simple units of the subdivision of the Venetian piede. In fact the Franciscus Patavinus virginal shows the use of the local unit of measurement in many other aspects of its design which have not been shown here. But the dimensions and balance point of the keyplank (ie. of the jointed board from which the keys were cut), the string scalings, the angling of the strings, the dimensions of the blocks from which the boxslide was made, etc. were all based on the use by Patavinus of the Venetian oncia. The dimensions of the Patavinus virginal show the use of simple units of the Venetian oncia in a manner that is particularly simple and clear. The dimensions of other instruments sometimes involve slightly more complicated numbers, and may involve subdivisions of the oncia, soldo or pollice into thirds, sixths and twelfths, as well as the more usual halves and quarters. Some of these are illustrated in the examples discussed below.

Table 2

Dimensions of baseboard without the case sides and mouldings.

Polygonal virginal by Franciscus Patavinus, 1552.

Museo Correr, Venice.

-	Measured dimension	Nominal d	
	mm	mm	once
Length:	1622	1622.9	56
Width:	359*	362.3	12½
Case left of the keywell:	348	347.8	12
Angled left side:	183	-	(6.31)
Angled left back:	862	-	(29.74)
Back:	304	304.3	10½
Angled right side:	555	_	(19.15)
Case right of the keywell	l: 565	565.1	19½
Keywell:	709	710.0	24½
Keywell projects:	116	115.9	4
Maximum case height	174	173.9	6

<sup>\*</sup> The more-or-less unaltered length of the keywell braces indicates that the wood of the baseboard has shrunk and that this measurement was probably originally about 362mm.

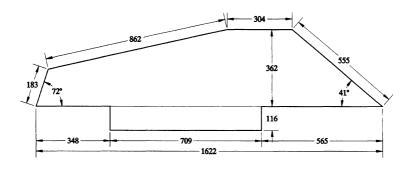


Fig. 3. Measured dimensions in mm of the baseboard without the case sides and measured angles at the front corners.

Polygonal virginal by Franciscus Patavinus, 1552

Museo Correr, San Marco, Venice

$$\tan 72^{\circ} = 3.07 \cong 3 = \frac{6}{2}$$
  $\tan 41^{\circ} = 0.869 = \frac{12.5}{14.4} \cong \frac{12.5}{14.5}$ 

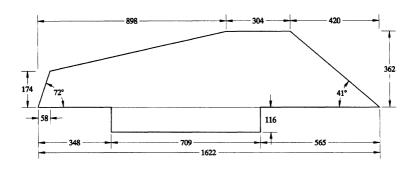


Fig. 4. Measured dimensions in mm of the baseboard without the case sides and measured angles at the front corners. Polygonal virginal by Franciscus Patavinus, 1552. Museo Correr, San Marco, Venice.

At the front left-hand corner:

174mm = 6.004 once
58mm = 2.001 once

420mm = 14.49 once
898mm = 30.99 once
304mm = 10.49 once

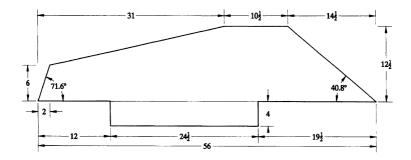


Fig. 5. Baseboard dimensions without the case sides measured in the Venetian oncia = 28.98mm, showing the front corner angles calculated from these.

Polygonal virginal by Franciscus Patavinus, 1552.

Museo Correr, San Marco, Venice.

$$\tan 71.6^{\circ} = 3 = \frac{6}{2}$$
  $\arctan \frac{6}{2} = 71.6^{\circ}$ 

$$\tan 40.8^{\circ} = 0.862 = \frac{12\frac{1}{2}}{14\frac{1}{2}}$$
  $\arctan \frac{12\frac{1}{2}}{14\frac{1}{2}} = 40.8^{\circ}$ 

The virginal by Franciscus Patavinus was clearly designed using the Venetian oncia of length 28.98mm. The baseboard measurements make this particularly obvious, and also show that the various angles were drawn, not by using a protractor, but by drawing the diagonal of a rectangle with sides which were a simple number of Venetian once in length. The position and length of the long diagonal side at the rear left-hand side of the instrument was drawn by joining the end of the near left-hand sloping side and a point on the rear of the baseboard which was 31 once in from the left end. Hence the irregular pentagonal shape of the baseboard arises from a series of orthogonal measurements, perhaps drawn out on a jointed plank that was originally 56 once (4½ Venetian piedi) long by 12½ once wide. The close agreement between the measured angles at the front corners of the baseboard and the angles calculated theoretically from the orthogonal components of the sides used to construct them is a further confirmation of the method used by Franciscus to construct the baseboard.

Working in reverse in those instruments where the centre in which they were built is not known it is possible to use the angles at the front corners to guess what the measurements used to construct them was, and from this to make an initial guess at the length of the unit of measurement. This will be illustrated in the examples below. In harpsichords the tail angle was normally constructed in a similar way, and using this angle to guess at the orthogonal components of the angle used to construct it can enable one to make an initial guess at the length of the unit of measurement used in the design and construction of all of the rest of the instrument.

### A VIRGINAL BY MARCUS SICULUS

A very fine sixteenth-century Italian virginal signed: '· MARCVS · SICVLVS · FACIEBAT · MDXXX ·', with vinework arabesques at the ends of the signature.<sup>17</sup> is to be found in the Benton Fletcher Collection of Early Keyboard Instruments housed in Fenton House. Hampstead in London. At first glance, except for the keywell scrolls, the virginal by Marcus Siculus is superficially similar to many Venetian virginals (see a schematic representation of the keywell section in Fig. 6). However, it is clear from the measurements of its case and baseboard given in Table 3 that the Venetian oncia was not used in its design. But if the instrument was not made in Venice how can we find the length of the oncia used by Siculus, and from this determine the centre where it was made and therefore where Siculus lived and worked? Out of the continuum of possibilities for the value of the oncia used by Siculus, some method is needed first of all to make an educated guess at a rough value of the length of the oncia he used, and then to refine this further.

<sup>&</sup>lt;sup>17</sup> Raymond Russell, Catalogue of the Benton Fletcher Collection of Early Keyboard Instruments at Fenton House, Hampstead, (Faber and Faber, London, 1957; revised London, 1969) 11. Russell casts doubt on the reliability of the signature on this instrument, but I can see no reason to question it. The instrument bears the accession number FEN/I/5. I would like to thank Mimi Waitzman, of the Benton Fletcher Collection, for her permission to examine and measure this instrument, and for her help in carrying out my examination.

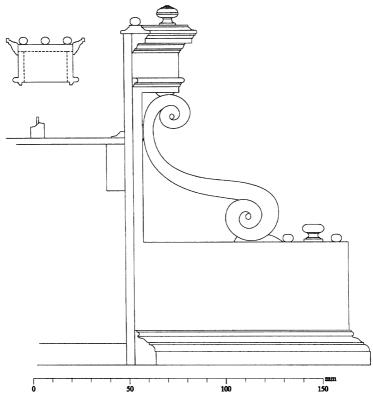


Fig. 6. A schematic representation of the keywell section, bridges and jackrail.

Polygonal virginal by Marcus Siculus, 1550.

Fenton House, Hampstead, London, Acc. N° FEN/I/5.

Table 3.

Measurements in mm of the baseboard without the case sides
Polygonal virginal by Marcus Siculus, 1550
Fenton House, Hampstead, London, Acc. N° FEN/I/5

The same of the sa			
Length:	1269	Back:	275
Width:	3321/2	Angled right side:	396
Case left of the keywell:	242	Case right of the keywell:	296
Angled left side:	143	Keywell:	731
Angled left back:	752½	Keywell projects:	111

Left-hand corner: 54 parallel to the front; 130 perpendicular to the front Corner angle =  $67\frac{1}{2}$ °

Right-hand corner: 216 parallel to the front;  $332\frac{1}{2}$  perpendicular to the front Corner angle =  $57^{\circ}$ 

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The process of determining the unit of measurement used by Siculus begins by looking at the geometry of the near left-hand corner. The tangent of the angle at this corner gives the ratio of the sides used to construct the corresponding angled side of the instrument.

For this virginal the measured angle at the near left-hand corner is 67%. Therefore:

$$\tan 67\%^{\circ} = 2.414$$

Also the lengths of the sides forming this angle (see Table 3) were measured and found to be 130mm and 54mm. Thus  $\frac{130}{54} = 2.407$ , a value which, as expected, is close to the tangent of the angle there.

A quick glance at a slide-rule 18 shows that 6/2.5 = 2.400 and suggests that the lengths of these two sides might have been designed to be 6 once and  $2\frac{1}{2}$  once. This suggests that 130 mm = 6 once so that there would be  $\frac{130}{6} = 21.67 \text{ mm/oncia}$  and that 54 mm = 2.5 once so that there would be  $\frac{130}{2.5} = 21.6 \text{ mm/oncia}$ . At the other corner of the instrument the measured angle is  $57^{\circ}$  and the tangent of this angle is therefore tan  $57^{\circ} = 1.540$ . The sides making up this angle have measured lengths of  $332\frac{1}{210} = 1.540$ . The sides making up this angle have measured lengths of  $332\frac{1}{210} = 1.58$  which, again as expected, is close to the value of the tangent there. A further glance at the slide rule shows that these are both close to  $\frac{15\frac{1}{2}}{10} = 1.55 \text{ suggesting that the two sides were designed to be <math>15\frac{1}{2} \text{ once } (= 332\frac{1}{2} \text{mm})$  and 10 once (= 210 mm). These all suggest a length for the oncia which can then be used for the other measurements of the instrument. The calculation of the size of the oncia are shown in Table 4 below:

<sup>&</sup>lt;sup>18</sup> The ratio here is fairly simple and the size of the components in local units used to make it up are fairly obvious. But when the ratios are more complicated, as they are at the right-hand corner of this instrument for example, then I know of no other better method of determining the two numbers that give rise to the ratio involved than using a slide rule. In fact the initial use of a slide rule to determine the ratio of the lengths of the component sides of the triangle making up the corner is essential to the analytical process of determining the unit of measurement used to design and construct the instrument. By setting the slide rule to the value of the ratio determined either from the tangent or directly from the measured lengths of the components used to make up the diagonal side, and then looking for the simplest numbers that make up this ratio, it soon becomes clear what these component lengths are in the local unit of measurement. A circular slide rule is particularly convenient for carrying out this procedure.

Table 4

The calculation of the local unit of measurement
Polygonal virginal by Marcus Siculus, 1550
Fenton House, Hampstead, London, Acc. N° FEN/I/5

Mea	surement in mm		Local unit		Length of
Component of left corner parallel to front:	54	=	2½ once	⇒	21.60
Component of left corner perpendicular to front:	130	=	6 once	⇒	21.67
Length of front:	1269	=	59 once	$\Rightarrow$	21.508
Baseboard width:	332½	=	15½ once	⇒	21.451
Case left of the keywell:	242	=	11¼ once	$\Rightarrow$	21.511
Back:	275	=	12¾ once	$\Rightarrow$	21.569
Case right of the keywell:	296	=	13¾ once	$\Rightarrow$	21.527
Keywell:	731	=	34 once	$\Rightarrow$	21.500
Keywell projects:	111	=	5½ once	$\Rightarrow$	21.484
Back at the right:	215	=	10 once	$\Rightarrow$	21.500
Back at the left:	778	=	36¼ once	$\Rightarrow$	21.462
Maximum case sides height:	171½	=	8 once	⇒	21.438
Total:	4605	=	214½ once	Average:	21.502mm

This is very close to the value of the *oncia* for Sicily/Palermo<sup>19</sup> where one palmo had a length of 257.8mm giving an *oncia* of 21.483mm (the difference is only 0.09%) or, using other sources for Palermo in Sicily,<sup>20</sup> the *oncia* had lengths which varied between the narrow limits of 21.483mm and 21.611mm.<sup>21</sup> The measurements of the baseboard of the 1550 Siculus virginal are shown in Figure 7 in millimetres in the top part of the diagram and in units of the Sicilian *oncia* in the bottom part of the diagram.

<sup>&</sup>lt;sup>19</sup> L. Malvasi, La metrologia italiana ne' suoi scambievoli rapporti desunti dal confronto col sistema metrico-decimale, (Fratelli Malvasi, Modena, 1842-44).

<sup>&</sup>lt;sup>20</sup> See Appendix 2 at the end of this paper.

<sup>&</sup>lt;sup>21</sup> An *oncia* of length near 20.17mm based on a *palmo* = 242mm seems also to have been used in Sicily (see Appendix 2 at the end of this paper).

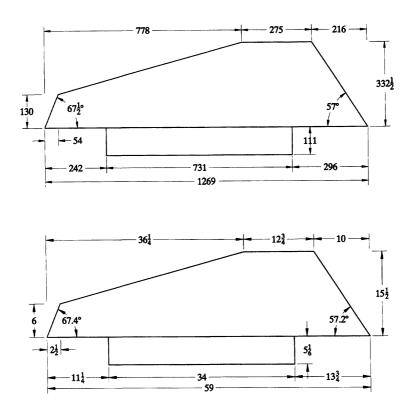


Figure 7. Measured angles in degrees and dimensions in mm (above), and nominal angles and measurements in Sicilian once (below) of the baseboard.

1 oncia = 21.502mm.

Polygonal virginal by Marcus Siculus, 1550. Fenton House, Hampstead, London, Acc. N° FEN/I/5.

Clearly the virginal was made in Sicily, probably in Palermo, using the Sicilian *oncia*. But then the name SICVLVS means 'from Sicily', so that the region in which the maker was working was really staring us in the face the whole time!!

The design of the instrument naturally did not stop with the baseboard and case height. The measurements of the string scalings are shown in Table 5 below<sup>22</sup> and are plotted in the graph of Figure 8. Here it is clear

<sup>&</sup>lt;sup>22</sup> These have all been corrected for a modern re-pinning of the bridges to compensate for a sideways movement of the strings caused by case distortion resulting from the string tension.

that the string lengths have a simple Pythagorean design based on  $f^1 = 20$  once from middle  $f^1$  to  $f^3$ , and that the lengths of the f strings for the part of the compass below  $f^1$  were also designed by Siculus using simple whole numbers of Sicilian once. The figure shows the basis of the string scaling design of this instrument in a particularly graphic way.

Table 5.

The original string scalings after correction.
Polygonal virginal by Marcus Siculus, 1550.
Fenton House, Hampstead, London, Acc. N° FEN/I/5

	String Length				
	Measured	Nom	inal		
	mm	mm	once		
$f^3$	107	107.5	5		
$c^3$	143				
$f^2$	215	215.0	10		
$\mathbf{c}^2$	301				
f¹	431	430.0	20		
$c^1$	575				
f	794	795.6	37		
c	969				
E/G#	1042				
D/G#	1070				
F	1077	1075.1	50		
C/E	1100				

The analysis of the case geometry and dimensions, and of the string scalings of the virginal by Marcus Siculus therefore provides a sort of internal consistency and proof of the validity of the method used to find the length of the unit of measurement of the maker and, in turn, of the centre in which the instrument was made. It also shows the usefulness of the method to our understanding of how the string scalings (and other features such as the dimensions of the keyplank from which the keylevers were cut, the keyplank balance-pin line, the plucking points of the f notes, all not shown here) were designed. But clearly it is possible with totally unsigned and anonymous instruments to carry out the same method to enable the determination of the unit of measurement and from the city or region in which the maker who designed the instrument lived and worked.

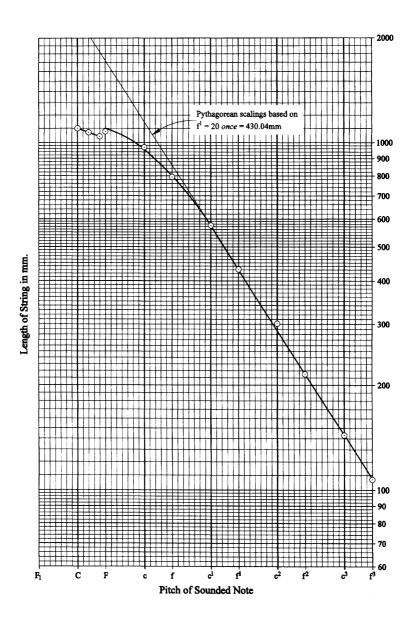


Fig. 8. The string scalings.

The straight line indicates Pythagorean scalings based on  $f^1 = 20$  Sicilian once = 430.04mm.

Polygonal virginal by Marcus Siculus, 1550.

Fenton House, Hampstead, London, Acc. N° FEN/I/5.

For example, using this method the calculation of the unit of measurement used in the design of an anonymous polygonal virginal (MS-60) in the Händelhaus in Halle<sup>23</sup> resulted in the conclusion that it was also built using an oncia of 21.5mm. This immediately suggested first, that the instrument was made in Sicily and second, that Siculus might also have been the maker of this instrument. The instrument in Halle is larger and the string scalings suggest that it was probably designed to sound a tone lower than the Siculus virginal in Fenton House in London. Comparison of the mouldings and the construction methods and materials showed that, although the unit of measurement used not only for the baseboard and case sides but also for the keyboard and string scalings was clearly the same, many of the other features were totally different. The mouldings were different both in their details and also in their general style. The handling of the case framing, the woods used and the jackrail support system are totally different in the two instruments, making it highly unlikely that they are actually by the same maker. Nonetheless it is still important that the maker of Halle MS-60 can be said also to have lived and worked in Sicily where, although they occur in instruments by different makers, two pitches a tone apart must have coexisted in a way similar to that of most of the other major centres in Italy.

Another example of the use of this method involves the instruments of Ignazio Mucciardi, about whom there is also no biographical information. A similar analysis of the unit of measurement used in the design and construction of the instrument in private possession in Salerno about 40 km. southeast of Naples<sup>24</sup> and attributed by me to Mucciardi,<sup>25</sup> and of the single-manual harpsichord in the Museo

<sup>&</sup>lt;sup>23</sup> See: Konrad Sasse, Katalog zu den Sammlungen des Händel-Hauses in Halle. 5. Musikinstrumentensammlung – Besaitete Tasteninstrumente, (Händel-Haus, Halle an der Saale, 1966) 28-9. I would like to express my thanks to Christiane Rieche who allowed me to examine this instrument, and to Stephan Ehricht who gave assistance in many ways including taking the moulding shapes of this instrument for me.

<sup>&</sup>lt;sup>24</sup> A detailed report on this single-manual harpsichord prepared by me in 1997 is held by the Padri Redentoristi, Convento di Pagano, Salerno. The instrument is believed to have belonged to St Alfonso, founder of the Padri Redentoristi. It was bought by him and was in his possession at the time of his death in 1780.

<sup>&</sup>lt;sup>25</sup> This attribution is based on the similarity in the construction methods, such as the use of diagonally-placed soundboard wood, the use of wedge-shaped pieces of bone in the ebony inlay of the sharps, the use of a panelled nameboard with inlaid decoration, the size of the bridge-, hitch- and tuning-pins, and upon the similarity in the shape of the decorative mouldings and the natural key arcades. I have no doubt that Mucciardi made the Salerno harpsichord. Only one harpsichord, in the Museo Nazionale degli Strumenti Musicali in Rome, is signed by Mucciardi and is referred to in footnote 26. At least three other instruments can be attributed to Ignazio Mucciardi on a similar basis. (contd)

Nazionale di Strumenti Musicali in Rome among others,<sup>26</sup> shows that these instruments were built using the Neapolitan *oncia*. This strongly implies that Mucciardi must have lived and worked in or near Naples, in the area where the Neapolitan *oncia* was being used. The fact that the Salerno harpsichord was owned by Sant'Alfonso, who founded the order of the Padri Redentoristi and who died in 1780 (the year in which the harpsichord in Rome was built) suggests that Mucciardi must indeed be from Naples, or possibly from Salerno. Mucciardi is a very common Neapolitan surname, and an archival search for biographical details of Mucciardi would, based on the information I have found from an analysis of the unit of measurement used in these instruments, have to begin in Naples or the surrounding area. A recent article published by Francesco Nocerino on harpsichord building in Naples<sup>27</sup> identifies a Pasquale

<sup>(25</sup> contd) The other instruments are also all single-manual instruments and are all built in the same style. The instrument in the collection of Dr Rodger Mirrey, Paddington, London, England has a compass of G<sub>1</sub>,A<sub>1</sub> to f<sup>3</sup>, and has a keyboard with boxwood naturals and with black and white sharps decorated in the same manner as the Salerno and the Rome instruments. The key arcades have not survived on this instrument. It also has a soundboard constructed with a sloping grain, and with a similar internal construction. It uses the same unit of measurement as the Rome and Salerno harpsichords (the owner holds a copy of a report by me analysing the unit of measurement used in the design of this instrument). Another harpsichord attributed by me to Mucciardi is in the Museum of Cultural History in the Smithsonian Institution, Washington, D.C. (Inv. No. 326,903). This instrument bears the false signature 'Johannes[sic] Antonius Baffo Venetus F MDLXXXI'). It also has a compass of G<sub>1</sub>,A<sub>1</sub> to f<sup>3</sup> and similar construction characteristics. Another instrument by Mucciardi is a single-manual bentside spinet in the Musikinstrumenten Museum in Berlin (See: Dagmar Droysen-Reber and Horst Rase, 'Historische Kielklaviere bis 1800. Beschreibung der Instrumente, Teil I', Kielklaviere. Cembali, Spinette, Virginale, General editor Dagmar Droysen-Reber, (Staatliches Institut für Musikforschung Preußischer Kulturbesitz, Berlin, 1991) Cat. No. 2216, pp.171-4). This instrument also has a compass of G1,A1 to f3. The Berlin spinet is not ascribed to Mucciardi in the new Berlin catalogue, but many features of its construction and decoration are clearly the same as those usual on the other Mucciardi instruments, such as the white wedge-shaped inlay in the top of the sharps, the panelled nameboard inlaid with black and white decoration, etc. From the information available in the Berlin catalogue it is also clear that the same size of oncia was used in its construction as in the Salerno, Rome and Mirrey harpsichords.

<sup>&</sup>lt;sup>26</sup> This instrument is not listed in Maria Luisa Cervelli, 'Per un catalogo degli strumenti a tastiera del Museo degli Antichi Strumenti Musicali', *Accademie e Biblioteche d'Italia*, 44, № 4-5 (1976) 305-43, but see Maria Luisa Cervelli, *La Galleria Armonica*, (Istituto Poligrafico e Zecca dello Stato, Rome, 1994) 279. This instrument was restored by me in 1980, exactly 200 years after it was built and signed 'Ignazio Mucciardi nipote del ? − − ? fecit 12 Giugno 1780'.

<sup>&</sup>lt;sup>27</sup> Francesco Nocerino, 'Arte cembalaria a Napoli. Documenti e notizie su costruttori e strumenti napoletani', *Ricerche sul '600 napoletano. Saggi e documenti 1996-1997*, (Electa Napoli, Naples, 1998) 85-109.

Mucciardi who was active in Naples in September of 1780. It seems highly likely therefore that Ignazio and Pasquale Mucciardi were both active in Naples in the same period and that they were probably related. The signature on the Rome instrument which reads 'Ignazio Mucciardi nipote del? —? fecit 12 Giugno 1780' suggests further that the illegible part of the signature might read 'Pasquale',  $^{28}$  and that Ignazio was the grandson or, more likely, the nephew<sup>29</sup> of Pasquale Mucciardi. Clearly without the determination of the unit of measurement used in these instruments it would be impossible to know where to begin a search for information about Ignazio Mucciardi, and indeed it would not have resulted in the knowledge that there was a harpsichord-building tradition in the Mucciardi family in Naples.

# A HARPSICHORD DATED 1631 BY STEFANO BOLCIONI, FLORENCE, IN THE YALE UNIVERSITY COLLECTION OF MUSICAL INSTRUMENTS, NEW HAVEN, CONN.

An unusual harpsichord in the Yale University Collection of Musical Instruments<sup>30</sup> bears the signature 'STEFANVS BOLCIONIVS PRATENSIS F MDCXXXI F' written in ink in Roman capitals on the lower back part of the nameboard. Below this in small cursive script is a second signature '1631 Stefanus · Bolcionius · Pratensis fecit'.<sup>31</sup> The compass is now C to f<sup>3</sup> chromatic, but was originally the common C/E to f<sup>3</sup> chromatic compass, and the original c<sup>2</sup> scalings were 262/263mm. Figure 9 shows a schematic representation of the cheek section and of the keywell scrolls of this harpsichord.

<sup>&</sup>lt;sup>28</sup> See footnote 26 above. The word 'Pasquale' would fit perfectly into the amount of space occupied by the illegible part of the signature with the spacing of the handwriting of the rest of the inscription.

<sup>&</sup>lt;sup>29</sup> In Italian the word *nipote* can mean either grandchild or nephew/niece.

<sup>&</sup>lt;sup>30</sup> This instrument bears the Yale University of Musical Instruments catalogue number 4889.72. My thanks to Richard Rephann, curator of the Collection, for his kind help and co-operation in my examination of this instrument.

<sup>&</sup>lt;sup>31</sup> Although it seems unusual that the instrument should be signed twice, both signatures are similar to those on other Bolcioni instruments. The one in Roman capitals is similar to that on the Russell Collection harpsichord, and the one in cursive script is similar to the signatures on the virginals by Bolcioni in Leipzig (see footnotes 39 and 41), Munich (see Hubert Henkel, Besaitete Tasteninstrumenten, (Erwin Bochinsky, Frankfurt-am-Main, 1994) Catalogue Number 1907-9231, pp.106-8), and in Rome (see Louisa Maria Cervelli, 'Per un catalogo degli strumenti a tastiera del Museo degli Antichi Strumenti Musicali', Accademie e Biblioteche d'Italia, 44, N° 4-5 (1976) Inv. No. 1764, pp.318-9 and La Galleria Armonica (Istituto Poligrafico e Zecca dello Stato, Rome, 1994) p.218. I can see no reason to doubt that both of the signatures on the Yale harpsichord were made by anyone other than Bolcioni.

The most unusual aspect of this harpsichord is that it has two nuts on the wrestplank. The nut nearest to the player serves the right-hand choir of strings and is slightly higher than the nut further from the player which carries the left-hand choir of strings. The two nuts are very close together in the treble and indeed one of them is partly sliced away in order that the two can be separated by the correct amount. Further down in the bass the two nuts gradually diverge until, at the lowest note, there are several centimetres separating them. The near, higher nut has scallops cut out of it to allow the strings of the far nut to pass by unimpeded to their tuning pins. The tuning pins are not arranged as normal, but are widely separated and the rear row of pins tunes the left-hand (what would normally be the long) choir of strings. The strings therefore diverge from the nut towards the tuning pins instead of remaining parallel as is more usual.<sup>32</sup>

The scalings of this harpsichord produced as a result of this unusual arrangement of the two nuts are given in Table 6:

Table 6

Scalings in mm of the original C/E to f<sup>3</sup> state
(the present state has been ignored).

Single-manual harpsichord by Stefano Bolcioni, Florence, 1631.
Yale University Collection of Musical Instruments, Catalogue No. 4889.72.

	Left-plu	icking 8'	Right-plucking 8'		Comparison o	of string lengths <sup>33</sup>
	String F	Plucking	String I	Plucking	Nominal	Florentine
	Length	Point	Length	Point	mm	sol <b>di</b>
$f^3$	97	45	96	28	96.4	3½
$c^3$	122	57	129	44		
$\mathbf{f}^2$	194	74	194	63	192.7	7
$\mathbf{c}^2$	262	86	263	79		
$\mathbf{f}^{1}$	399	102	399	100	399.2	141/2
$c^1$	532	113	530	114		
f	783	127	776	131	784.6	28½
с	1039	137	1017	140		
F	1472	150	1450	153	1472.9	53½
C/E	1486	152	1497	155		

It is clear from Table 6 that the intention of the maker was to equalise the scalings of the two 8' registers by a correct positioning of the two nuts. In the middle of the compass around f<sup>1</sup> Bolcioni achieves both

<sup>&</sup>lt;sup>32</sup> The reason for this unique arrangement is almost certainly that, by making the strings diverge as they leave the two nuts, more space can be given between the pins in the near nut and the scalloped cut-outs for the strings of the second nut further from the player. Otherwise the pins on the near nut would have had to have been placed in a weak position right on the edge of the scalloped cut-outs.

 $<sup>^{33}</sup>$  Here, anticipating the results found below, the string scalings and other measurements have been expressed in units of the Florentine *soldo* = 27.52mm (see footnote 38).

scalings and plucking points which are essentially the same for both sets of 8' strings. Clearly string scalings and plucking points were important to Bolcioni and aspects which occupied a significant role in his (and that of most other harpsichord and virginal maker's) overall design. And this design was clearly based on the Florentine soldo.

The measurements of the baseboard and case height are given in Table 7 for this instrument:

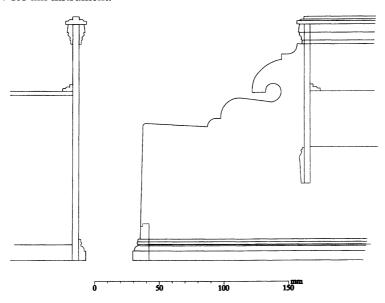


Fig. 9. Schematic representation of the cheek section and the keywell scrolls.

Single-manual harpsichord by Stefano Bolcioni, Florence, 1631.

Yale University Collection of Musical Instruments, Catalogue No. 4889.72.

Table 7.

Baseboard dimensions and case height.
Single-manual harpsichord by Stefano Bolcioni, Florence, 1631.
Yale University Collection of Musical Instruments, Catalogue No. 4889.72.

	Measured dimension in mm		
Length	: 1827		
Width	778½		
Cheek	422		
Tail	212		
Tail angle	: 75°		
Component of tail perpendicular to the spine	205		
Component of tail parallel to the spine	: 56		
Case height	184		
$\tan 75^{\circ} = 3.$	$732 \cong 3.75 = \frac{7\frac{1}{2}}{2}$		

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# ANALYSIS OF UNIT OF MEASUREMENT USED IN THE CONSTRUCTION OF THE YALE BOLCIONI SINGLE-MANUAL HARPSICHORD:

The procedure for determining the unit of measurement used to construct this harpsichord begins with the measurement of the angle of the tail, which was found to be 75°. The tangent of this angle is tan  $75^{\circ} = 3.732 \cong 3.75 = \frac{71/2}{2}$ . This suggests that the two sides of the triangle that were used to construct the tail angle are  $7^{1/2}$  soldi<sup>34</sup> and 2 soldi which, mathematically, would form an angle of  $75.07^{\circ}$ . This angle is very close to the measured angle and well within the error of measurement. Measurement in millimetres of the length of the two sides constituting the orthogonal components of the tail side gives an approximate estimate of the size of the soldo which can then be applied to the other measurements of the baseboard, keyboard, wrestplank, string scalings, and all of the other parts and design features of the instrument. A summary of the measurements of the baseboard and case height in soldi is given in Table 8:

Table 8.

The calculation of the local unit of measurement.

Single-manual harpsichord by Stefano Bolcioni, Florence, 1631.

Yale University Collection of Musical Instruments, Catalogue No. 4889.72

asurement Local in mm unit	Length of soldo
56 = 2 soldi	<b>⇒</b> 28.0
205 = 7½ soldi	<b>⇒</b> 27.33
1827 = 67 soldi	<b>⇒</b> 27.27
778½ = 28½ soldi	<b>⇒</b> 27.32
422 = 15½ soldi	<b>⇒</b> 27.23
184 = 6¾ soldi	<b>⇒</b> 27.26
3472½ = 127¼ soldi	Average: 27.29mm
	in mm unit  56 = 2 soldi  205 = 7½ soldi  1827 = 67 soldi  778½ = 28½ soldi  422 = 15½ soldi  184 = 6¾ soldi

$$\tan 75.19^{\circ} = 3.75 = \frac{7\frac{1}{2}}{2}$$
  $\arctan \frac{7\frac{1}{2}}{2} = 75.07^{\circ}$ 

<sup>&</sup>lt;sup>34</sup> It will be shown below from the size of the unit of measurement that Florence is the centre in which the instrument was built. The Florentine *braccio* was divided into 20 *soldi*, and not into *once* as found in some other centres.

These measurements are shown in the diagram in Figure 10 where the actual measurements in millimetres are shown on the left, and the measurements in units of the local measurement are shown on the right.

For Florence Johann Georg Krüniß<sup>35</sup> mentions the use of the *bavelle* which is clearly equal to a *palmo* or half a Florentine *braccio*.<sup>36</sup> As the *braccio* was divided into 20 *soldi*, the *bavelle*, like the *palmo*, must have had a length of 10 *soldi*. The calculated length of the *bavelle* given by Krüniß is 273.41mm, so that the *soldo* must therefore have had a length of 27.341mm. This seems clearly to be the unit being used by Bolcioni (the difference between this and the unit found here for the Yale harpsichord is only 0.2%).

A further look at Appendix 2 giving the units of measurement used in the various centres in Italy during the historical period shows that in Florence the *braccio*, divided into 20 units, had a length according to Angelo Martini<sup>37</sup> of 551.202mm. Hence the *soldo* had a length of:

$$\frac{551.202}{20} = 27.560$$
mm.

The length of the *braccio* is also variously given as 550.6371,<sup>38</sup> giving a *soldo* of:

$$\frac{550.6371}{20}$$
 = 27.53mm.

These are both close (error  $\cong 0.8\%$ ) to the length of the *soldo* found here and seem to confirm that the instrument was indeed made in Florence.

<sup>&</sup>lt;sup>35</sup> Johann Georg Krüniß, Öconomische Encyklopädie oder allgemeines System der Staats-, Stadt-, und Landwirtschaft, in alphabetischer Ordnung, 15 (Joseph Georg Traßler, Brünn, 1788) pp. 519-22. These are given as 1440-th parts of the Paris pouce, and were converted into millimetres by me using the millimetre length of the pouce given by Colonel Cotty, Aide-Mémoire, p.896 (see footnotes 10 and 38). My thanks to John Koster for pointing out this source to me.

<sup>&</sup>lt;sup>36</sup> The plural form of *braccio* is irregular in Italian and changes gender so that *il braccio* in the singular becomes *le braccia* in the plural.

<sup>&</sup>lt;sup>37</sup> See: Angelo Martini, Manuale di metrologia, (E. Loescher, Turin, 1883; reprint Editrice Edizioni Romane d'Arte, Rome, 1976) 206. Martini is one of the few authors to give the length of the braccio and soldo before the standard of length in Florence was re-defined by legislation passed on 2 July, 1782.

<sup>&</sup>lt;sup>38</sup> Colonel Cotty, Aide-Mémoire a l'usage des officiers d'artillerie de France, 2 (Paris, 1819) 896-7 gives the length for the Florentine braccio da terra divided into 20 soldi as 550.3671mm.

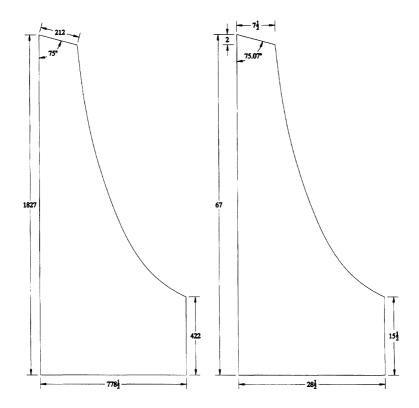


Fig. 10. Baseboard measurements in millimetres (left)
and
in units of the Florentine soldo used by Bolcioni = 27.29mm (right).
Single-manual harpsichord by Stefano Bolcioni, Florence, 1631.
Yale University Collection of Musical Instruments, Catalogue No. 4889.72.

# A VIRGINAL DATED 1641 BY STEFANO BOLCIONI, FLORENCE, IN THE MUSIKINSTRUMENTENMUSEUM, UNIVERSITY OF LEIPZIG

The virginal by Bolcioni in the Musikinstrumentenmuseum at the University of Leipzig<sup>39</sup> is in a fairly ruinous state, but very interesting as a result of never having been restored in modern times.<sup>40</sup> This instrument is signed 'Stefanus bolcionius Pratensis 1641' written in cursive script in ink on the back of the namebatten.<sup>41</sup> The compass is C/E to f<sup>3</sup> with a broken short octave with split D/F#, E/G#, and then split g#/ab and g#1/ab¹, giving it altogether 54 notes. The  $c^2$  string scaling is 328mm, apparently based on 12 Florentine soldi = 328.09mm (see footnote 35), or, equivalently, of f¹ = 18 Florentine soldi.

It is a rectangular virginal with the right rear corner missing from the rectangle so that, placed in its outer case, the empty space provides a toolbox in the normal way. Here the only triangle which can be used to make an initial estimate of the unit of measurement is that of this rear toolbox space. A summary of the original measurements of the baseboard and case height is given in Table 9.

<sup>&</sup>lt;sup>39</sup> This instrument does not bear a Leipzig Musikinstrumentenmuseum catalogue number as it is on loan from the Leipzig Museum für Kunsthandwerk. My thanks to Ezster Fontana and Klaus Gernhardt of the Musikinstrumentenmuseum of the University of Leipzig for their help and co-operation in allowing me to examine this instrument. Please note that this instrument is not listed among the other instruments by Bolcioni in Donald H. Boalch, *Makers of the Harpsichord and Clavichord*, 1440-1840, (3rd Edition, Clarendon Press, Oxford, 1995) pp.248-9. I do not want here to digress into the intricate reasons why both this instrument and the virginal in Munich (see footnotes 31 and 74) were thought by Hubert Henkel not to be by Bolcioni. As mentioned in footnote 31, I see no reason to doubt the signatures nor the authenticity of either of these two instruments for all of the usual reasons – the workmanship, materials, mouldings, unit of measurement, etc. are similar for all of these instruments.

<sup>&</sup>lt;sup>40</sup> The jacks, for example, have beautifully-cut plectra which may well be original eighteenth-century French raven quills!

<sup>&</sup>lt;sup>41</sup> The signature is incorrectly given as 'Stefanus Colcionius Pratensis 1641' by Hubert Henkel in Kielinstrumente. Katalog des Musikinstrumentenmuseums der Karl-Marx Universität Leipzig, Vol. 2 (VEB Deutscher Verlag für Musik, Leipzig, 1979) p.112.

Table 9.

Baseboard dimensions and case height. Rectangular virginal by Stefano Bolcioni, Florence, 1641. Musikinstrumentenmuseum, University of Leipzig (On loan from the Leipzig Museum für Kunsthandwerk).

	Measured dimension mm
Length:	1592
Length of rear spine:	1244
Baseboard width:	424
Short right-hand end:	136
Case left of the keywell:	384
Keywell:	710
Case right of the keywell:	498
Keywell projects:	117
Component of toolbox side along the spine:	348
Component of toolbox side along the right side:	287
Angle of toolbox side:	50½°
Height of case sides:	210
$\tan 50\%^\circ = 1.213 \cong 1.214 = \frac{1234}{1204}$	$\frac{348}{207} = 1.2125$

$$\tan 50\frac{1}{2}^{\circ} = 1.213 \cong 1.214 = \frac{12\frac{3}{4}}{10\frac{1}{2}}$$
  $\frac{348}{287} = 1.2125$ 

## ANALYSIS OF THE UNIT OF MEASUREMENT USED IN THE CONSTRUCTION OF THE LEIPZIG BOLCIONI RECTANGULAR VIRGINAL:

The procedure for determining the local unit used to construct this virginal begins with the measurement of the toolbox angle at the rear right-hand corner of the instrument. The tangent of this angle is tan  $50\frac{1}{2}$ ° = 1.213  $\cong$  1.214 =  $\frac{12.75}{10.5}$  and this suggests that the two sides of the triangle that form the toolbox are 1234 soldi and 101/2 soldi which, mathematically, would form an angle of 50.53°. This angle is very close to the measured angle of 501/2°. Measurement in millimetres of the length of the two orthogonal components of the toolbox side gives an approximate estimate of the size of the soldo. Table 10 shows the calculation of the unit of measurement used in the Bolcioni rectangular virginal based on the assumption that the sides of the toolbox at the rear right-hand side of the instrument were constructed geometrically using lengths of 123/4 soldi and 101/2 soldi.

Table 10.

Calculation of the local unit of measurement.

Rectangular virginal by Stefano Bolcioni, Florence, 1631.

Musikinstrumentenmuseum, University of Leipzig
(On loan from the Leipzig Museum für Kunsthandwerk).

M	easurement Local in mm unit		Length of soldo
Toolbox angle component parallel to spine:	348 = 12¾ soldi	⇒	27.29
Toolbox angle component perpendicular to spine:	287 = 10½ soldi	⇒	27.33
Total length:	1592 = 58¼ soldi	$\Rightarrow$	27.33
Length of rear spine:	$1244 = 45\frac{1}{2} $ soldi	$\Rightarrow$	27.34
Baseboard width:	424 = 15½ soldi	$\Rightarrow$	27.35
Short right-hand end:	136 = 5 soldi	⇒	27.0
Case left of the keywell:	384 = 14 soldi	$\Rightarrow$	27.43
Keywell:	710 = 26 soldi	⇒	27.31
Case right of the keywell:	498 = 18¼ soldi	$\Rightarrow$	27.29
Keywell projects:	117 = 4½ soldi	$\Rightarrow$	27.53
Height of case sides:	210 = 7½ soldi	⇒	27.39
Total:	5950 = 217¾ soldi	Average:	27.34mm

These measurements are shown in the diagrams of Figure 11 where the actual measurements in millimetres are shown in the top diagram, and the measurements in units of the Florentine soldo are shown in the diagram at the bottom. The value of the length of the soldo found for this instrument is very close both to that found for the Yale Bolcioni single-manual harpsichord (error 0.04%) and to the reference values already discussed for the previous instrument (see footnotes 37 and 38). This excellent agreement helps to confirm both that the instruments are made by the same maker and that their design is based on the Florentine soldo.

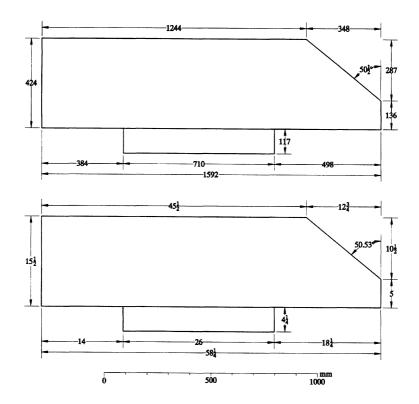


Fig. 11. Baseboard measurements in millimetres (above) and in Florentine soldi = 27.34mm (below).

Rectangular virginal by Stefano Bolcioni, Florence, 1641.

Musikinstrumentenmuseum, University of Leipzig (On loan from the Leipzig Museum für Kunsthandwerk).

# FURTHER WAYS IN WHICH THE UNIT OF MEASUREMENT WAS USED IN THE DESIGN OF A VIRGINAL OR HARPSICHORD

It is not surprising that the local unit was used in the design and execution of virtually every aspect of the construction of an instrument, so that its use can be recognised in many aspects other than the baseboard and case height measurements. Some of these can, in turn, be used to extract the unit of measurement used in the design of the instrument when this is otherwise unknown.

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The unit of measurement must also apply to the width and sides of the keyplank<sup>42</sup> (ie. of the outer measurements of the jointed board from which the keys were cut), the balance line marked on the keyplank (ie. the distance of the balance line at the outside edges of the keyplank ignoring the added natural touchplates and the arcades), the angling of the strings, the scalings of either the c or of the f notes, etc. It is usually not at all clear what the unit of measurement is that will give simple numbers for the measurements of all of these different aspects of the construction of the instrument, and it is a stab-in-the-dark procedure to try to determine the unit for all of these different measurements in any situation, such as with a rectangular virginal or with a clavichord, where it is not possible to rely on the geometrical methods outlined above. Another hint is necessary in order to arrive at a rough value of the *oncia*, soldo or pollice that can then be refined as was done with the geometrical method described above.

During my analysis of a number of instruments I have noticed, at least with many of the virginals built in Venice and in centres where the *oncia* had a length of about 30mm, that the width of the blocks used to make the boxslide have a width of ½2 of an *oncia*, or of 5 *linee*, regardless of what the absolute size of the local *oncia* might be<sup>43</sup> (see Fig.12). In the Italian tradition the boxslide is made up of a number of flat blocks of wood, each with two shallow recesses in them inside which the jacks move. The blocks are glued together so that the lateral spacing of the pairs of recesses corresponds to the lateral spacing of the ends of the keylevers, which is often also a simple division of the local unit of measurement. 44 Care seems to have been taken in Venice and the other Italian centres using an *oncia* with a size of about 30mm to make the thickness of each of these blocks exactly 5 *linee*. Thus 24 such blocks would have a thickness of 24 x ½2 = 10 *once*. The choice of a total width

<sup>&</sup>lt;sup>42</sup> The angling of both the balance line and the rear of the keyplank of a virginal effectively provides two further angles and measurements and, from them, possible estimates of the unit of measurement for the instrument being studied. This aspect is not elaborated here but provides yet another example of how an initial estimate of the unit of measurement could be obtained.

<sup>&</sup>lt;sup>43</sup> In most of the North-Italian centres the unit of measurement is usually around 27 to 32mm. However in Rome, Naples and Sicily, and in such northern centres as Genoa and Mantua, for example, where the *oncia* was only 18 to 21mm, the blocks of the boxslide would have to be more than ½2 of an *oncia* thick, otherwise the strings would be placed uncomfortably close to one another and to the jacks. *See* Appendix 2 at the end of this paper. The use of the *oncia* in the design of the blocks of the boxslide registers in Brescia and Milan where the size of the unit of measurement was greater than 32mm is elaborated briefly below.

<sup>&</sup>lt;sup>44</sup> For example, in many sixteenth- and early seventeenth-century Venetian virginals, the width of the 50-note C/E to f<sup>3</sup> keyboard plank was designed to be 25 *once*. See footnote 47.

of 10 once for 24 register blocks may be a throwback to an earlier period when keyboard compasses were often F,G,A – f<sup>3</sup>, four octaves without F# and G#, with 47 notes which would have required 24 register blocks.<sup>45</sup> For simplicity in the design this was given a width of 10 once. Because the strings are normally parallel to the jackslots in the slightly-angled boxslide, the width of 24 register blocks can be measured simply by measuring the width of 24 complete pairs of strings (omitting one string at one end of the string band or the other for the usual C/E to f<sup>3</sup> instruments) in a direction perpendicular to the strings. Hence measuring the width of the string band may be enough to determine the unit of measurement in the small number of instruments where this width was designed by the maker to be 10 once.

Figure 12 shows a drawing of the bass end of the boxslide register of the 1552 Marco Jadra virginal<sup>46</sup> to illustrate how its construction is based on the Venetian *oncia*. In this case the keyboard was designed to be 25 *once* wide<sup>47</sup> so that the 50 keytails of the 50-note C/E to f<sup>3</sup> compass were each exactly ½ *oncia* wide, and so that the successive blocks containing two jackslots had a lateral spacing of precisely 1 *oncia*, and a thickness of ½ of an *oncia*.<sup>48</sup>

Needless to say the geometry of the virginal boxslide registers is not always as simple as that found in the Venetian instruments. Clearly when the local unit of measurement is markedly different from about 30mm the maker is forced to design the width of his string band and registers with other dimensions in order to avoid either an unnecessarily narrow or unnecessarily wide string band. Gianfrancesco Antegnati, working in Brescia (where 1 *oncia* = 39.62mm), made the total width of 24 pairs of jackslots equal to  $7\frac{1}{2}$  Brescian *once* (39.62mm x  $^{7.5}$ %4 = 12.38mm per jackslot). Also Annibale de' Rossi, working in Milan (where one *oncia* or *pollice* = 36.265mm), gave the width of 48 strings (24 jackslots) a width of 8 *pollici*  $^{49}$  so that each boxslide block had a thickness of  $^{8}$ %4 =  $^{1}$ % of a *pollice* (12.09mm). The latter measurement for the register-block width when

<sup>&</sup>lt;sup>45</sup> In fact almost all of the surviving virginals of Gianfrancesco Antegnati working in Brescia in about 1550 have or originally had this F,G,A to f <sup>3</sup> compass. Antegnati uses 24 register blocks each with 2 slots in them, and leaves the second jackslot between the ends of the F and G keylevers unused.

<sup>&</sup>lt;sup>46</sup> Illustrated in Francis W Galpin, *Old Instruments of Music*, (Methuen, London, 1910) p.124, plate XXIII. My thanks to Hélène La Rue for her co-operation and help in allowing me to examine this instrument.

<sup>&</sup>lt;sup>47</sup> The width of the keyplank of this virginal is  $724\text{mm} = \frac{724}{28.98} = 24.98$  once, obviously meant to be 25 once.

<sup>&</sup>lt;sup>48</sup> See further my article, 'Marco Jadra. A Venetian harpsichord and virginal builder?', *Gedenkschrift für Kurt Wittmayer*, to be published in 1999 and edited by Silke Berdux referred to already in footnote 9.

<sup>&</sup>lt;sup>49</sup> In Milan the subdivision of the *piede* was called either the *oncia* or the *pollice* (the thumb).

expressed in mm is fortuitously almost exactly the same for Milan as that resulting from the use of  $\frac{5}{12}$  of a Venetian *oncia* found above.

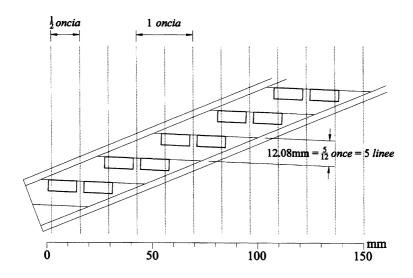


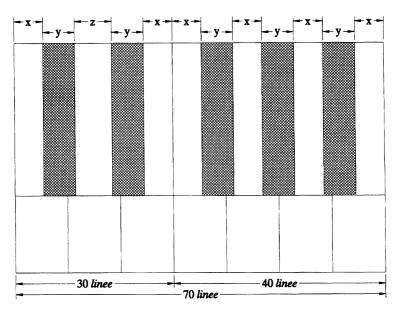
Fig. 12. The boxslide register of the 1552 Marco Jadra polygonal virginal in the Pitt Rivers Museum, Oxford, No 1948.1β1.

1 Venetian oncia = 28.98mm.

Staying with the 1552 virginal by Marco Jadra in the Pitt Rivers Museum, Oxford, it is clear that the Venetian *oncia* was used in the design of a number of the other aspects of the keyboard. Here the 50 notes of the C/E to f<sup>3</sup> compass have a width of 25 *once* (see also footnotes 44 and 47). Hence the lateral spacing of the keylever tails is just ½ *oncia* per key. The 30 natural notes also have a width of 25 *once* so that each natural is  $\frac{25}{30} = \frac{5}{6} = \frac{10}{12}$  of an *oncia* wide, or each natural is 10 *linee* wide, and one octave with 7 natural keys is 70 *linee* in width. The sharps, the c, e, f, g, a, and b keytails, and the d keytails can be shown then to have widths of 6 *linee*,  $5\frac{1}{2}$  *linee* and 7 *linee* respectively. One octave is therefore composed of the width of the sharps = 5 x 6 *linee*, plus the width of the c, e, f, g, a, and b keytails = 6 x  $5\frac{1}{2}$  *linee*, plus the width of the d keytail = 7 *linee*, giving a total width of 70 *linee*, the same as that calculated using the natural fronts (see Fig.13).

The 25 once width of the keyboard gives rise to a 3-octave span (the width of 21 naturals) of  $25 \times \frac{25}{30} = 17\frac{1}{2}$  once. Since the Venetian oncia = 28.98mm (see footnote 10), this gives rise to a 3-octave span of  $17\frac{1}{2} \times 28.98 = 507$ mm, exactly equal to the measured 3-octave span, and a value

near to that found on many other Venetian stringed keyboard instruments which are clearly using this measurement and division of the keyboard.



 $x = 5\frac{1}{2}$  linee, y = 6 linee and z = 7 linee

Fig. 13. A typical division of one octave in the keyboard of a sixteenth-century Venetian harpsichord or virginal when the total width of the 50-note C/E to  $f^3$  compass = 25 once

One Venetian piede = 347.76mm; one oncia =  $\frac{1}{12}$  piede = 28.98mm and one linea =  $\frac{1}{12}$  oncia = 2.415mm

On the other hand there is also a number of Venetian instruments such as the Franciscus Patavinus virginal and the 1568 virginal also by Marco Jadra in the Victoria and Albert Museum, London<sup>50</sup> which have a keyplank that was designed to be  $24\frac{1}{2}$  once in width instead of the 25 once as above. This gives rise to a three-octave span of  $24\frac{1}{2}$  x  $\frac{21}{30}$  x 28.98mm = 497mm, a value also found to be close to the measured

<sup>&</sup>lt;sup>50</sup> See: Howard Schott, Catalogue of Musical Instruments. Volume 1 – Keyboard Instruments. Victoria and Albert Museum, (Victoria and Albert Museum, London, 1985) Museum No. 155-1869, pp. 24-5. My thanks to James Yorke, Assistant Curator of Furniture and Woodwork at the Victoria and Albert Museum for his co-operation and assistance in allowing me access to this instrument.

value.<sup>51</sup> Marco Jadra is not alone in occasionally using different measurements for the keyplank width of his instruments, giving rise to different consequent measurements of the 3-octave span. Clearly the three-octave span of an instrument is *not* a characteristic of a maker since the same maker sometimes used different values for this measurement. The use of the words *Stichmaß* and *standard measure*<sup>52</sup> for this width is clearly inappropriate since the width of the octave, of 3-octaves or the total width of the keyplank cannot in any way be considered standard or characteristic of a maker. Rather, the different sixteenth-century Venetian makers using the common 50-note C/E to f<sup>3</sup> compass, for example, practically all begin the design of their instruments by making the total keyplank width either 24½ or 25 *once*. Therefore the measured 3-octave spans of 479mm and 507mm resulting from these keyplank widths are characteristic of Venice and not of the individual makers working there.

Clearly the string scalings themselves were designed using simple values of the local unit of measurement, and a number of examples of this have already been seen incidentally in the consideration of some of the instruments discussed above. These string measurements were often designed using whole integers of the unit of measurement and not integers plus complicated fractions. This suggests that the makers were using simple, easy-to-remember numbers, and were not necessarily concerned with the subtleties of taking the strings as close as possible to their breaking point by choosing complicated fractional numbers in the design of their string scalings. In Venice, for example, the instruments of Ioannes Celestini, Dominicus Pisaurensis, Benedetto Floriani, etc. use either integral or half-integral numbers of the Venetian oncia as the basis of their string-scaling design. I have been able to show<sup>53</sup> that two of the instruments of Marco Jadra, a virginal of 1568 in the Victoria and Albert Museum<sup>54</sup> and the other a virginal of 1552 in the Pitt Rivers Museum, Oxford<sup>55</sup> were separated in pitch by a tone (major second) or, using my usual convention, by R+2. In this case the design of the instruments separated in pitch by this amount is particularly elegant and simple since

<sup>&</sup>lt;sup>51</sup> The keyboard based on a keyplank width of 24½ Venetian *once* would not have keyfronts and keytails and octaves divided in a simple way like that of the 25 *once* keyboards. However it would be a simple matter of using a geometrical projection of the 25 *once* design to give a keyboard with a width of 24½ *once* and with all of its other width dimensions in proportion both at the keyfronts and at the keytails.

<sup>&</sup>lt;sup>52</sup> See Howard Schott, in the reference given in footnote 50.

<sup>&</sup>lt;sup>53</sup> See my article, 'Marco Jadra. A Venetian harpsichord and virginal builder?', Gedenkschrift für Kurt Wittmayer, to be published in 1999 and edited by Silke Berdux already referred to in footnote 9.

<sup>&</sup>lt;sup>54</sup> See: Howard Schott, Catalogue of Musical Instruments. Volume 1 – Keyboard Instruments. Victoria and Albert Museum, (Victoria and Albert Museum, London, 1985) Museum No. 155–1869, pp.24–5

<sup>55</sup> See footnote 46.

the f<sup>2</sup> scalings were based by Jadra on 9 *once* and on 8 *once*, the Pythagorean ratio between the string lengths of two notes a tone apart being simply %!

Another aspect of the use of the unit of measurement in the investigation of the history of an instrument can be illustrated from the analysis of the design and construction of the anonymous single-manual Italian harpsichord in the Royal College of Music in London, Catalogue N° RCM 175. Calculation of the unit of measurement used in its construction in a way similar to that used for the Yale University Bolcioni harpsichord makes clear that the instrument was designed and built using the Neapolitan oncia = 21.736mm.<sup>56</sup> The instrument was modified a number of times before it was given its present state.<sup>57</sup> Many features, such as the moulding on the top of the present nut, the use of separate upper and lower guides instead of boxslides, the shape of the moulding on the outside edges of the upper guide, the construction and guiding system used for the keyboard, etc. are typical of those found on instruments by the Florentine makers Bartolomeo Cristofori and his pupil Giovanni Ferrini. But is there evidence that the Florentine soldo was used in the construction of any of the components of the present state of this instrument which would help to link it to Florence and a Florentine workshop?

The present two registers have a moulding on their outside edges which is characteristic of the work of Cristofori and Ferrini, and seems to be from their workshop. Hence, as these two both worked in Florence, the registers should have been constructed using the Florentine *soldo*. To check this the spacing of the jackslots along the register was measured.

Figure 14 shows a graph of the jackslot spacing of the front register of RCM 175. Here the distance from the spine of the instrument to the edge of each jackslot is plotted against the note sounded by the jack whose jackslot is being measured. The more-or-less uniform spacing of the jackslots gives rise to a straight-line plot whose mathematical characteristics can be calculated using normal statistical analysis.

The usual linear regression analysis by the method of least squares gives a correlation coefficient for this data of r = 0.9999936 indicating a very good fit of the measured data to a straight line. The calculated slope of the line is m = 13.7675mm/jackslot with a standard deviation error of only 0.0101 (0.07%).<sup>58</sup>

<sup>&</sup>lt;sup>56</sup> According to a number of different sources in Appendix 2 at the end of this paper the *oncia* in Naples had a length close to 21.81mm.

<sup>&</sup>lt;sup>57</sup> These modifications are outlined in an unpublished restoration report by John Barnes held by the Royal College of Music.

<sup>&</sup>lt;sup>58</sup> My thanks to Orestis Papasouliotis of the STATLAB Statistics Laboratory at the University of Edinburgh for his help in the determination of the accuracy of these results. Here, to calculate the standard deviation error, it was assumed that the error of measurement was 0.1mm, that the error in marking out and cutting the register slots by the re-builder was 0.1mm so that the total error in the position of each slot was 0.2mm.

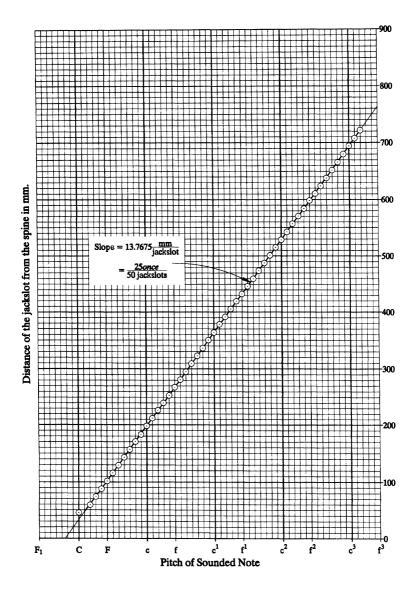


Fig.14. Spacing of the jackslots using the Florentine soldo Anonymous Italian single-manual harpsichord, Naples, c.1650 Royal College of Music, London, Cat. No. 175

This slope = 13.7675mm/jackslot is equivalent to 0.50005 soldi/jackslot, based on the Florentine soldo of 27.532mm found in reference

tables. This therefore appears to be a spacing of exactly 50 jackslots in 25 soldi:

$$13.7675 \frac{\text{mm}}{\text{jackslot}} = \frac{25 \text{ soldi}}{50 \text{ jackslots}}$$

Using this to calculate the soldo gives:

1 soldo = 
$$\frac{13.7675 \times 50}{25}$$
 mm = 27.54mm

This compares with the value given by Colonel Cotty<sup>59</sup> for the *braccio* divided into 20 *soldi* of 550.64mm, of 1 *soldo* =  $\frac{550.64}{20}$  = 27.532mm. This is only 0.01% different from that estimated here and strongly suggests that the register slots were indeed cut out by designing them to be exactly ½ of a Florentine *soldo* apart.

It would be an incredible coincidence, therefore, if the instrument was not altered in Florence. The use of the Florentine soldo and braccio in the design of the registers, and the similarity of the construction features of the added and altered parts to those normally found on the instruments of Cristofori and Ferrini gives additional strength to the argument that the instrument was indeed given its present final state by one of these two builders who both worked very much in the same tradition. This is then further confirmed when the Florentine soldo is applied to the dimensions of the keyplank from which the keyboard was cut (also made in the style of Cristofori and Ferrini), to the altered string scaling design, etc. all of which were clearly designed in simple units of the Florentine soldo. This, in addition to the many other characteristics, make it almost a certainty that one of these two makers had a hand in the re-working of this instrument.

Needless to say the size of the *soldo* found for this instrument reworked in Florence by Cristofori or Ferrini is the same as the *soldo* found for the two Bolcioni instruments built entirely in Florence and discussed earlier in this paper.

# LIMITATIONS OF THE METHOD AND WORDS OF CAUTION

The methods described above used to ascertain the unit of measurement are only one aspect of the determination of the centre of construction of a harpsichord or virginal, and only one aspect of establishing the maker of an anonymous instrument. In order to be certain of the authorship of an otherwise anonymous instrument it is

<sup>&</sup>lt;sup>59</sup> See Colonel Cotty, Aide-Mémoire a l'usage des officiers d'artillerie de France, 2 (Paris, 1819) 896-7.

necessary to compare such factors as the methods of workmanship, the materials, the case mouldings, the string scalings, and the unit of measurement used in the instrument's design and construction. One of these features on its own is not enough.

The use of the unit of measurement in this analysis does of course rely upon the accuracy and reliability of the sources from which the lengths of the units of measurement have been taken. Many of the sources are derivative and simply repeat the measurements given by earlier authors. The original need for the publication of these tables of measurements arose chiefly as a result of metrification imposed by law in the period between the Napoleonic invasion and the Unification of Italy, and the resulting need to relate the old units of measurement to the metre in the period in which modern Italy gradually took on a united nationhood. However by this time, and indeed during this period, legislation had changed the sizes of a number of the units of measurement somewhat from those used in the historical period of harpsichord building. For example in Tuscany, including Florence, the length of the braccio was altered as a result of legislation passed on 2 July, 1782.60 Also a law was passed on 6 April 1840 in Naples which increased the length of the palmo and the other local units of measurement by about 0.3338%, a small but significant amount.<sup>61</sup> Some sources published after 1840, such as Ludovico Eusebio<sup>62</sup> using the 'decimalised' palmo, and the anonymous author of the article 'Misure'63 in the Grande dizionario enciclopedico, give the later value of the length without taking into consideration the value before 1840. Most of the sources, however, even when published after 1840 give the pre 6 April 1840 value of the palmo and canna in Naples. It is therefore clear that great care has to be taken when using the published tables of measurements when making ascriptions based on them. This applies especially to the Southern area of Naples and Sicily which were sometimes separate and sometimes united in the 'Regno delle due Sicilie' during the historical period. An instrument which apparently uses the Sicilian measurement may well have been made in Naples using the Neapolitan unit, and vice-versa.

Are we to trust the surprises thrown up as a result of the use of these tables? A good example of one such surprise is provided by a fine anonymous single-manual harpsichord, part of the collection of the

<sup>&</sup>lt;sup>60</sup> See Angelo Martini, *Manuale di metrologia*, (E. Loescher, Turin, 1883; reprint Editrice Edizioni Romane d'Arte, Rome, 1976) 206.

<sup>&</sup>lt;sup>61</sup> See Giovanni Gandolfo, Tavole di ragguaglio ovvero prontuario di compiuti fatti di pesi, misure e monete legali italiane, (Naples, 1860) pp.12-17.

<sup>&</sup>lt;sup>62</sup> Compendio di Metrologia Universale e Vocabolario Metrologico, (Unione Tipografico Editrice Torinese, Turin, 1899; reprint by Forni Editore, Bologna, 1967).

<sup>&</sup>lt;sup>63</sup> Anonymous author, *Grande dizionario enciclopedico*, 12 (Unione Tipografico-Editrice Torinese, Turin, 1970) p.626.

Civici Musei Veneziani d'Arte e di Storia, in the Ca' Rezzonico in Venice.<sup>64</sup> This is a separate inner instrument in an outer case, and is extremely long having a spine measurement of 2505mm. The compass is  $G_1, A_1$  to  $c^3$ , and it has an elegant-looking keyboard with skunktail sharps and inlaid ivory decoration in the ebony-topped naturals. One of the most characteristic features of this harpsichord is the way in which all of the case and jackrail mouldings and the nut and bridge have been ebonized, thus accentuating the line and form of the instrument (see Fig.15). Unlike many Italian harpsichords which have one register fixed in position so that its jacks are always plucking its set of strings, both registers are movable and can be operated using a sophisticated hidden stop-lever mechanism. Incorporated into the design of the keyblocks on either side of the keylevers are small ebony buttons which operate iron rocker bars connected to the registers. Moving the ebony buttons from side to side engages and disengages the corresponding register. This system has been carefully worked into the design of the harpsichord and is also an individual and characteristic feature of this instrument.

Beginning with the angle of the tail of this harpsichord in the usual way described above it is clear from the baseboard and case-height measurements that the maker of this instrument was using an *oncia* with a length close to 29.37mm. Although close to the Venetian *oncia* of 28.98mm it is clear that the Venetian unit does not apply to this instrument. A number of the other measurements of the instrument such as the width and height of the internal core of the jackrail, the distance from the top of the soundboard liner to the top of the case sides, the height of the lower outside case moulding, the keyplank dimensions and the position of the balance pin line on the keyplank, etc. can also be shown to have been designed and measured out using this same *oncia* unit. The length of the *piede* with 12 *once* used by the maker of this instrument would therefore have been 12 x 29.37mm = 352.44mm.

The only important centre in Italy which used a unit of length near to this measurement during the period in which this instrument was built was Urbino. The *piede* in Urbino had a length near 353.5mm,<sup>65</sup> making the *oncia* there 29.46mm only 0.3% different from the value obtained

<sup>&</sup>lt;sup>64</sup> A study of this harpsichord was made as part of the same project for the Civici Musei Veneziani d'Arte e di Storia as noted in footnote 11. The Museo Correr holds an unpublished report entitled *Cembalo italiano anonimo ad una tastiera dalla Ca' Rezzonico* by me on this instrument.

<sup>65</sup> Giovanni Croci, Dizionario universale dei pesi e delle misure in uso presso gli antichi e moderni con ragguaglio ai pesi e misure del sistema metrico, (The Author, Milan, 1860), the anonymous author of the Tavole di ragguaglio fra le nuove e le antiche misure . . . della Repubblica Italiana pubblicate per ordine del Governo, 2 (Milan, 1809), L. Malvasi, La metrologia italiana ne' suoi scambievoli rapporti desunti dal confronto col sistema metrico-decimale, (Fratelli Malvasi, Modena, 1842-44) and Luigi Pancaldi, Raccolta ridotta a dizionario di varie misure antiche e moderne coi loro (contd)

deriving the length of the unit of measurement from the instrument. This therefore suggests that Urbino may have been the centre in which this harpsichord was built. However, no stringed keyboard instrument maker is known to have worked in Urbino. Was the instrument therefore really made in Urbino? It has many individual and highly characteristic features such as the ebonized mouldings and bridges, the use of mother-of-pearl and ivory in the panelling of the nameboard, and the ingeniously-hidden stop-lever mechanism, all of which suggest that it came from a tradition with clearly-defined attributes not normally found in any other tradition. The possibility that this instrument is a unique example of harpsichord making in Urbino, perhaps characterised by these features, is at least a sufficient cause for instigating archival work in Urbino to see if there is any evidence for stringed keyboard instrument making there in the seventeenth century.

The geometrical method of estimating the local unit of measurement from the tangent of the corner angle of a polygonal virginal or the tail angle of a harpsichord fails completely if the angle is  $45^{\circ}$ . In this case the tangent (tan  $45^{\circ} = 1$ ) does not suggest two unique small simple numbers from which the local unit can be estimated: here any two numbers are possible, all of which have a ratio to one another of 1! This seems at first like a great failing of the method, but so far I have encountered this problem only once,  $^{66}$  and then only for the right-hand front corner angle of a polygonal virginal – the left-hand front corner was not  $45^{\circ}$  and enabled an estimate of the local unit to be made.

In all cases discussed so far it has been the tangent of the corner or tail angle that was used to estimate the local unit of measurement and indeed was that used in the design of the instruments being studied. However in the single-manual harpsichord by Onofrio Guarracino dated 165167 the

<sup>(65</sup> contd) rapporti alle misure metriche . . ., (Sassi, Bologna, 1847) give values of the piede in Urbino between the narrow limits of 353.37mm to 353.793mm, so that the oncia had a value close to 29.46mm.

<sup>66</sup> This occurs in the 1568 polygonal virginal by Marco Jadra in the Victoria and Albert Museum, London. See: Howard Schott, Catalogue of Musical Instruments. Volume 1 – Keyboard Instruments. Victoria and Albert Museum, (Victoria and Albert Museum, London, 1985) Museum No. 155-1869, pp. 24-5. This instrument is discussed in detail in my article 'Marco Jadra. A Venetian harpsichord and virginal builder?', Gedenkschrift für Kurt Wittmayer, to be published in 1999 and edited by Silke Berdux, and referred to already in footnote 9.

<sup>&</sup>lt;sup>67</sup> This instrument is in private possession in Rome. The date of the instrument is not entirely clear: it is either 1651 or 1657. My thanks to Andrea di Maio for bringing this instrument to my attention and for supplying me with information about it. This instrument is not listed along with the other instruments signed by Guarracino in Donald H Boalch, *Makers of the Harpsichord and Clavichord*, 1440–1840, (Third edition, edited by Charles Mould, Clarendon Press, Oxford, 1995) pp.343–6.

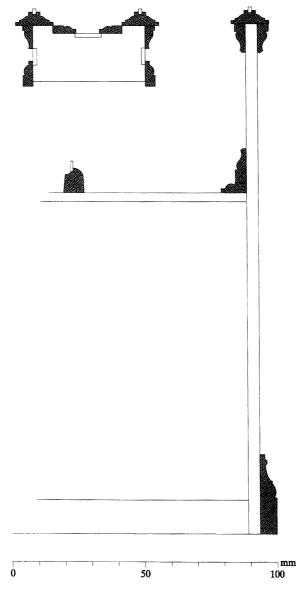


Fig.15. Schematic representation of the case mouldings, the jackrail section, and the bridge section at the position of the c² bridge pin.

The ebonized sections are indicated with shading.

Anonymous single-manual harpsichord, ?Urbino?, c.1630.

Ca' Rezzonico, Venice.

tail angle is clearly 30°. The tangent of 30° is 0.57735 . . ., an irrational number not composed of the ratio of two small simple numbers. However the sine of 30° is exactly 0.5000, suggesting that the two sides used by Guarracino to construct the tail angle were the side of the tail itself and the component of this side opposite the tail angle. Indeed this is found to be the case and the lengths of these two sides suggest an *oncia* = 21.61mm close to the *oncia* used in the other instruments by Guarracino.<sup>68</sup> Clearly then it is not always the tangent of the angle that was used, and the reader must accept that the sine and perhaps the cosine<sup>69</sup> were also used. Nonetheless the method of estimating the local unit of measurement remains the same.

Another potential limitation of this method is the inaccuracy of normal handworking methods. The method is relatively insensitive to this problem. With a large protractor it is possible to estimate the corner angles to within less than ½ of a degree. An error of ½° in an angle does not normally make enough difference to the value of the tangent for the usual tail or virginal corner angles in the range of  $30^{\circ} - 60^{\circ}$  to lead to the wrong estimate of the initial value of the ratio of the lengths of the two orthogonal sides of the triangle making up the angle. Hence the initial estimate of the unit of measurement is unaffected by a small error in the maker's construction, or the researcher's measurement, of this angle. However if there is a large error in the angle resulting from the handworking methods, then a false estimation of the unit of measurement can result. An example of this problem occurred in the analysis of an apparently well-made anonymous polygonal virginal in the collection of Marlowe Sigal of Boston, Massachusetts.<sup>70</sup> An analysis of the raw measurements of the lengths and corner angles of the baseboard of this virginal suggested initially that it was made in Florence. However the instrument is clearly of Venetian origin from the style and materials of its construction. But if it is assumed that the maker of this instrument

<sup>&</sup>lt;sup>68</sup> The study of this instrument and a number of other harpsichords that can be shown to be by Guarracino will form the subject of a paper currently in preparation.

<sup>&</sup>lt;sup>69</sup> This study, like many others involving an examination of the fruits of human endeavour, is scattered with pitfalls. The polygonal virginal in the Victoria and Albert Museum, London by Gianfrancesco Antegnati (Inv. No. 490-1899) has a measured angle of  $60^{\circ}$  at its front right-hand corner, suggesting that Antegnati may have been using the cosine of  $60^{\circ} = 0.5000$  to construct the right-hand sloping side. However, comparison with other instruments by Antegnati from which the size of the *oncia* that he was using can be calculated shows that the perpendicular and parallel components of the angled right-hand side have lengths of 9 and 5¼ *once*. Here it is fortuitous that tan  $59.75^{\circ} = \frac{9}{5.25} = 1.714$ . In other words, the fact that the measured angle was  $60^{\circ}$  (actually  $59.75^{\circ}$ ) does not, in this case, mean that the sides involved in the cosine of  $60^{\circ}$  were being used in its design. It is still the orthogonal components of the sloping side and therefore the tangent being used in the usual way.

<sup>&</sup>lt;sup>70</sup> I would like to express my thanks to Marlowe Sigal for his help in measuring this instrument prior to its analysis by me.

removed 3mm too much from the left corner during the finishing of the baseboard, then the angle at this corner changes, the components of the angled side change, the overall length changes and the distance of the bass end of the keywell to the left corner of the instrument changes. If the missing 3mm are added to all of these, then the calculations of the local unit of measurement used to construct this virginal give a clear indication that Venice was indeed the centre in which it was built. This is a good example of the blind use of only one method to assign a centre of construction or maker from only one of the many features of an instrument which must be invoked during the process of authentication.

The rough estimate of the unit of measurement obtained by assuming a width for one register block of \$\frac{5}{12}\$ of an *oncia* in virginals does give a unit that applies to the other measured lengths in a number of instruments. However, not surprisingly, it does not apply to all instruments and all makers. As mentioned above it does not apply in regions where the unit is considerably smaller or larger than 30mm. Therefore this way of determining the unit of measurement for rectangular virginals is, as suggested previously, only one method of approach in the determination of the unit of measurement.

The method of using the geometry of a corner angle of a virginal or the tail angle of a harpsichord described here appears to fail completely for rectangular instruments such as rectangular virginals and clavichords where there are no obvious corner angles to be used. However, because the lengths of the sides of the baseboards of such instruments were usually measured out in whole numbers of the local unit, my limited experience with such instruments so far suggests that the ratio of the sides of the baseboard itself can be used. When measured out in millimetres and when used in conjunction with the tangent of the angle of either diagonal of the rectangular baseboard, an estimate of the size of the unit of measurement can be obtained in the usual way although the numbers involved are clearly much larger than those found for the corner angles of virginals or for the tail angle of a harpsichord. Also, usually there are angled components in these instruments (such as the wrestplank, for example) which can be used in addition to give and initial estimate of the unit of measurement.

In addition a word of caution has to be added to allow for an occasional inability to distinguish two or more centres because their local units of measurement are either very similar or the same,<sup>71</sup> or because

<sup>71</sup> Both the piede manuale and the piede liprando with an once = 42.81mm were used, because of the political affiliations in the period of the Savoy, in both Genoa (Liguria) and Turin (Piemonte). Also there is a coincidental similarity in the oncia of the Genoese piede and the oncia of the Roman palmo mercantile both of which are close to 20.75mm.

they are in a simple proportion to one another.<sup>72</sup> Again additional features must be examined in order to establish the centre of origin of the instrument. Fortunately, however, the sizes of the units of measurement in the Italian Peninsula are quite widely spaced and spread over the range of about 18 to 58mm so that the determination of the unit of measurement leads to a clear conclusion about the region in which the instrument was built.

#### **CONCLUSIONS**

A knowledge of the unit of measurement has been shown here to provide one of the most important and potent methods for the analysis of the construction method and design used by makers of stringed keyboard instruments in the historical period. The determination of the size of the unit and the centre in which the unit of measurement was used can be invoked to suggest or confirm the maker of an instrument. Establishing the centre in which an instrument was constructed using this analysis can greatly narrow the field of possible makers from the large number with which one would otherwise be faced. The method can also be used, as with the Marcus Siculus and Ignazio Mucciardi instruments, to suggest the centre in which these makers, about whom no biographical information is otherwise available, lived and worked. The method can also suggest, as with the anonymous single-manual harpsichord in the Ca' Rezzonico in Venice, that harpsichords may have been built in centres such as Urbino, not previously recognised as locations in which stringed keyboard instruments were made.

It is of course important that, using the methods described here, the same result should be obtainable by any investigator. As with any scientific process the method for determining the unit of measurement used in the design and construction of Italian stringed keyboard instruments during the historical period described here is impartial and unbiased, being based on some of the simple geometrical methods and construction principles used by their makers. To that extent it does not involve any preconceptions or bias on the part of the investigator. And to that extent it is not dependent on who the investigator is or what his or her preconceptions or biases might be.

As suggested at the beginning of this paper a knowledge of the centre in which an instrument was made is basic to the understanding of musical practice in that centre. Although it has been recognised for some time

<sup>&</sup>lt;sup>72</sup> Reference has already been made in footnote 6 to a situation in which the centre of construction of an instrument is made uncertain because the units of measurement used in two cities are in the simple ratio of 3 to 4.

that Italian instruments are only superficially similar<sup>73</sup> there is still a strong tendency, based on the inability in the past to be able to distinguish regional styles in Italian stringed instrument keyboard construction, to consider a single tradition for the Italian peninsula as a whole. In fact, however, there are many regional traditions, each with their own design principles, construction methods, decorative styles, pitch standards, etc. These traditions changed and evolved over the 300-year historical period of harpsichord and virginal building in Italy, and the way that this happened is complicated by the alteration and re-use of older, out-of-date instruments (see footnote 73). Nonetheless it is clear that the time is now ripe for a study of the stringed keyboard instruments made on the peninsula based on a distinction and division according to these regional variations.

Any attempt to understand the stringing and pitch of Italian stringed keyboard instruments, for example, is doomed to failure unless the instruments studied are correctly grouped according to the region in which they were built. The fact that an instrument built in Florence and one built in Naples have the same scalings does not necessarily mean that they were designed to sound at the same pitch. Regional variations in pitch standards and stringing materials could strongly affect the pitch at which the two instruments were meant to sound even though their scalings are the same. Similarly two instruments from different centres with string scalings in the ration of 9 to 8 does not in itself mean that they were designed to sound at pitches a tone apart for similar reasons. Hence any study of Italian pitch and stringing practice is premature unless the centres of construction are known with certainty. Indeed because more than half of the surviving Italian stringed keyboard instruments are anonymous any such study would be inaccurate unless the additional information provided by the anonymous instruments, assigning their centre of origin correctly, is incorporated. The procedure outlined here provides a method for doing this and is fundamental to any such study, whether of pitch and stringing or of any other aspect of the design of such instruments.

Although the method outlined here is useful for all of the reasons indicated above, there is one further way in which its application is important. I want to show in a subsequent paper which will be published next year in this Journal that a knowledge of the size of the unit of measurement can be used as a powerful tool in the analysis and determination of the original state of an instrument that has been radically altered. In this case the analysis shows that the use of the unit of measurement, because it entered into the designer's mind in all aspects of

<sup>&</sup>lt;sup>73</sup> See John Barnes, 'The specious uniformity of Italian harpsichords', *Keyboard Instruments*, Edwin M. Ripin editor, (Edinburgh University Press, Edinburgh, 1971; reprint, Dover Publications, New York, 1977) 1-10.

the construction and design of the instrument, is essential to the determination of the original case length, the original scalings, the original dimensions and compass of the keyboard, the layout of the wrestplank and nut, and the position and angle of the lower belly rail. Without a knowledge of the unit of measurement used by the maker of this instrument the determination of the original state would have otherwise been impossible.

The implications of the use of the unit of measurement in instrument design are far-reaching. An extremely exciting prospect still requiring investigation is that the unit of measurement was the fundamental factor responsible for the regional variation in pitch found throughout the Italian peninsula. One of the most commonly-heard sounds during the historical period of harpsichord and virginal building would have been, not that of a plucked string, but that of a pipe in a pipe organ. The influence that church and chamber organs had on establishing the local pitch must have been extremely important. Since these organs were, like all other objects, made using the local unit of measurement, the size of the unit must have influenced the pitch of the organ. An 8' pipe in one centre would be a different length from an 8' pipe in another because of the difference in the local unit of measurement. To what extent this factor really affected local pitch is at least very interesting and certainly requires further investigation.

Perhaps the most interesting aspect of the method, however, is that it begins to give an idea of how the makers of these artefacts thought and how they worked. It enables us to enter into the minds of the instrument makers, and shows that they worked in a very pragmatic practical way. I have found no evidence whatsoever in the work that I have done so far in this field that the makers were in any way concerned with the use of the Golden Ratio or of the numbers that make up the Golden Series.<sup>74</sup> It is when one discovers that the blocks in the boxslide register of a virginal are each 5 *linee* in thickness, that the spacing of the jacks in a harpsichord register is exactly ½ *oncia*, that the keylevers comprising the 50 notes of the C/E to f<sup>3</sup> compass in Venetian instruments have a width of 25 *once* so

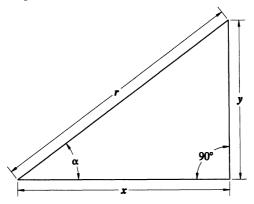
<sup>&</sup>lt;sup>74</sup> This is contrary to the evidence collected by Hubert Henkel, in the catalogue of the keyboard instruments from the Musikinstrumentensammlung of the Deutsches Museum in Munich (Besaitete Tasteninstrumente. Fachbuchreihe das Musikinstrument, Vol. 57 (Verlag Erwin Bochinsky, Frankfurt-am-Main, 1994). The evidence presented, incorrectly in my view, by Henkel is not based on the measurements of the instruments as designed by the makers: Henkel, in the Italian instruments, uses the outside case measurements and not the measurements of the baseboard without the case sides and, in the North-European instruments, the total height of the case adding the baseboard thickness to the height of the case side planks, etc. and then he seems, to me at least, to indulge in number crunching in order to make the results fit the numbers from the Golden Series.

that each keytail is exactly 6 lines in width and each natural is 10 lines in width (and that the width of the sharps and the spacing of the tails of the naturals are also based on a simple number of *linee*), that the tone separation in pitch of many Venetian instruments is based on the use of f<sup>2</sup> scalings of 9 *once* and 8 *once*,<sup>75</sup> etc. that one really starts to understand how makers thought. It is only then that that one begins to realise how simply and elegantly their instruments are designed.

#### APPENDIX 1

#### DEFINITIONS OF THE GEOMETRICAL FUNCTIONS

The geometry of the simple right-angled triangle is basic to the understanding of the design of Italian keyboard instruments. The definitions of the sine (sin), cosine (cos) and tangent (tan) of the angles of a right-angled triangle are based on the ratios of the lengths of the sides x, y and r in the diagram below:



The angle  $\alpha$  is measured in degrees and this angle, for baseboard corners of Italian stringed keyboard instruments, usually has a value between about 30° to 60°. Here by definition:

$$\sin \alpha = \frac{\gamma}{r} \qquad \cos \alpha = \frac{x}{r} \qquad \tan \alpha = \frac{\gamma}{x}$$
 and 
$$\arcsin \frac{\gamma}{r} = \alpha \qquad \arccos \frac{x}{r} = \alpha \qquad \arctan \frac{\gamma}{x} = \alpha$$

 $<sup>^{75}</sup>$  The ratio of the frequencies, or the inverse ratio of the lengths, of two notes a Pythagorean tone (major second) apart is 9/8.

Of these the most important factors involved in the determination of the unit of measurement used in the design of Italian stringed keyboard instruments are  $\tan \alpha$  and  $\arctan \frac{\gamma}{x} = \alpha$ . The actual values of the  $\tan$  and arctan can be calculated using a normal scientific calculator, they can be found in tables of the geometrical functions, or they can be read from any good scientific slide rule.

#### APPENDIX 2

### ITALIAN METROLOGY c.1500 TO 1800

An excursion into the field of historical Italian metrology is not for the faint hearted! It is a Pandora's box full of unexpected tricks waiting for the unwary. As the whole of the Italian peninsula gradually changed to the metric system during the nineteenth century, numerous works were published dealing with the conversion of the units of measurement of length, area, volume and currency into the new metric system which had been imposed by law first of all after the Napoleonic invasions of the North, and eventually after the Unification of Italy as a whole. The measurements given by the authors of these works are, however, not always the ones which were actually used in the Italian peninsula in the historical period of harpsichord building. In some areas the standards were changed in the period after that in which harpsichords were built but before the publication of the works on metrology. In Florence, for example, the unit of length was increased by a factor of 17/16 (61/4%) a result of legislation passed on 2 July, 1782, and also an increase of only 0.1% in Piemonte in 1818, and by 0.333% in Naples after 1841. It is therefore necessary to be sure that one is, in fact, applying the correct unit of measurement to an instrument in assigning to it its putative place of origin.

The other problem faced by a worker in this field is that there were various subdivisions of the palmo, piede and braccio. These were variously into 10, 12, 16, 20, etc. units and so it is clearly essential to understand how each of the units, whether the palmo, piede, braccio, oncia, pollice, etc., were subdivided. Although these were usually into 12 units, a division into 10 units was common in many of the towns in the Province of Emilia-Romagna. The latter division is not to be confused with the decimalization of the larger units introduced in some parts of Italy, for example in Sicily, during the nineteenth century. The division of the braccio was usually into 20 units, but divisions into 12, 16, 22, etc. were also known. When used for measuring cloth, the braccio was often divided in halves, quarters, eighths and sixteenths. The sub-division of the units of measurement used in the design of early keyboards instruments and

found in a number of important centres is given below and in tables 11 and 12 at the end of this Appendix.

## Cagliari and Sardinia:

1 canna = 10 palmi; 1 piede = 2 palmi = 12 once; 1 palmo = 6 once; 1 oncia = 12 punti [Dou, Eu]. (For the meaning of italicised abbreviations in square brackets see 'Authors consulted' below.)

#### Florence:

A law was passed in Florence on 2 July, 1782 which changed the length standard in Tuscany, as noted above, so that the value of the soldo = 29.18mm used from then until 1 July, 1861 is not valid for the historical period. In Florence and much of Tuscany 1 braccio = 2 palmi = 20 soldi = 12 crazie = 60 quattrini = 240 denari, so 1 palmo = 6 crazie = 10 soldi = 30 quattrini = 120 denari, and 1 soldo = 3 quattrini = 12 denari and 1 crazia = 1% soldi = 5 quattrini = 20 denari and 1 quattrino = 4 denari.

#### Milan:

1 braccio = 12 once or pollici = 144 punti = 1728 atomi = 20736 momenti [Cr, page 38].

# Naples:

A law was passed on 6 April, 1840 which increased the length of the palmo and other units of measurement in Naples and the surrounding area which came under the influence of the Kingdom of Naples by about 0.3338%, a small amount [see Ga, 1864]. Any sources such as Mal 1875 give the later value of the length without taking into consideration the value before 1840. In Naples 1 canna = 8 palmi = 96 once = 480 minuti, so 1 palmo = 12 once = 60 minuti = 600 punti, and 1 oncia = 5 minuti = 50 punti, hence the oncia is divided in 5 parts and not in 12!

#### Piacenza:

1 piede da legno = 12 once = 144 punti = 1728 atomi = 20736 minuti = 248,832 momenti = 2,985,985 scrupoli [Source: Cr page 39].

Piemonte (especially Turin, but valid throughout all of the smaller and larger centres in Piemonte):

14 once = 1 raso (braccio da panno); 8 once = 1 piede manuale. According to Mar, p.783, Eu, p.46 and others, the value of the piede was changed from 513.766mm to 514.403mm in 1818. Before 1818 (except for the period in which Napoleon dominated Savoy from 1798 to 1816 and therefore outwith the historical period of stringed keyboard instrument making) all of the other measurements were based on the piede legale, piede liprando or the raso with an oncia = 42.814mm.

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#### Rome:

There were three, probably four, basic sizes for the *oncia* in Rome. One, equal to about 18.617mm, was used for almost all of the normal measurements of objects, buildings, wood, etc. and was the basis of the *palmo romano, piede romano, braccio romano, passetto, passo*, and *canna architettonica*. Cloth appears to have been measured in units of two different *once*, equal to about 17.67mm and 20.75mm, and were the basis of the *palmo mercantile*, palmo da tela, braccio mercantile, braccio da tela, braccio da tessitore, canna mercantile, etc. The piede (close to 297.9mm) was normally divided into 16 units of the 18.617mm *oncia*, but it also appears that it was divided into 12 *once* giving another *oncia* of length near 24.82mm (used, for example, by Francesco Fabbri and Giambattista Boni both of whom worked in Rome). Dou says that the piede antico = 294.5mm was still in use in 1840. In Rome 1 piede = 1½ palmi = 16 once. 1 palmo = 12 once. 1 oncia = 5 minuti = 10 decimi, hence the oncia is divided in 5 or 10 parts but not in 12!

Tables 11 and 12 below give the sizes of the units of measurement current in all of the centres in which harpsichord and virginal makers were active in the historical period, as well as a few additional centres which were important culturally and commercially. These are arranged both according to the centre and according to the size of the oncia, soldo or pollice. These tables are also listed on the Russell Collection Website at http://www.music.ed.ac.uk/russell/metrology/. Also listed on this site for downloading are the complete databases from which these two tables were extracted. Text versions of these databases are also arranged according to the centre and according to the size of the unit of measurement, but are also available as working databases in several formats which can be manipulated by the appropriate data-base programmes in the usual way. The tables below are the condensation of the larger database which has about 2,500 entries. The tables below do not include any measurements given by the nineteenth-century sources for the period outwith the historical era of harpsichord building. Sources such as Co, Did, Cha and Kr which were actually published in the historical period are often listed separately in the tables below because of their obvious importance and, usually, accuracy.

#### **EQUIVALENT MEASUREMENTS**

In many of the regions of Italy, the units of measurement used in minor towns or centres were, for obvious reasons, equivalent to those of the particular region where they were located or to those of the nearby major centre. The reader should note that the towns given below used measurements which were the same as those of the region in which they were located or under whose domination they found themselves:

Ferrara Argenta, Cento, Comacchio, Codigoro and Pieve di Cento.

Florence Arezzo, Empoli, Livorno, Montepulciano, Pisa, Pistoia, Porto

Ferraio, Prato and Sienna.

Genoa Chiavari, La Spezia, Novi Ligure, Oneglia, Porto Plata, Savona and San Remo.

Milan Lodi, Monza, Pallanza and Treviglio.

Naples Acconza, Aci Reale, Aquila, Avellino, Bari, Barletta, Benevento, Brindisi, Caggiano, Cagniolo, Calabria, Campobasso, Caserta, Catanzaro, Cava, Cava, Chieti, Cosenza, Eboli, Fiano, Foggia, Gallipoli, Ischia, Isani, Lecce, Lucera, Mazzarà, Nocera, Nota, Potenza, Puglia, Reggio di Calabria, Rocca, Salerno, Taranto and

Teramo.

Palermo Caltanissetta, Campobasso, Catania, Catanzarro, Girgenti, Lipari, Marsala, Messina, Ragusa, Siracusa and Trapani.

Perugia Foligno, Gubbio, Narni, Spoleto and Terni.

Piacenza Bardi, Bobbio, Carpaneto, Fiorenzuola and Pellegrino.

Reggio Coreggio, Gualtieri, Luzzara, Reggio nell'Emilia, Reggiolo and Scandiano.

Rome Civitavecchia, Frosinone, Orvieto, Rieti and Viterbo.

Trento Riva di Garda and Tiarno.

Turin Alba, Asti, Biella, Cuneo, Ivrea, Mondovì, Pinarolo, Saluzzo, Susa and Vercelli.

Venice Asolo, Bassano, Belluno, Ceneda, Chioggia, Chions, Conegliano, Cristoglia, Gaiarine, Istria, Lugo, Mestre, Muggia, Portobuffolè, Portole, Prata, Ravenna, Rovigo, San Leonardo, Treviso, Trieste and Vicenza.

San Giorgio di Levenza and Portoguarro.

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Table 11

The length of the oncia, soldo or pollice in the Italian peninsula arranged according to the length of the unit

	m to man out	c charm,		Lamar I		The configuration of the control of	mg to the tengan of the time
Location	Unit	шш	Sub- unit	Sub- Length Divi- unit in mm sion	Divi- sion	Source	Notes
Rome	Piede antico	294.50	oncia	18.406	16	Did, Kr, Dou	Dou says that this piede was still in use in 1840.
Rome	Piede romano	297.00	oncia	18.563	16	Са, Еи	1 piede Romano = 16 once
The Marche <sup>76</sup>	6 Piede	335.10	oncia	18.617	18	Cr,EI,Dou,Pa,Ta2,Mal	1 piede da legname = $18$ once
Rome	Palmo (architettonico)	223.42	oncia	18.619	12	Cha, Did, Co, Or, Mal, Mar, Dou, Eu	1 palmo = 12 once = 120 decimi = $60$ minuti
Rome	Piede	297.90	oncia	18.619	16	Dou, Cr, Mal, Mar, Cr, Pa, Kr	1 piede = $1\frac{1}{3}$ palmi = 16 once
The Marche <sup>77</sup>	7 Palmo	223.42	oncia	18.619	12	Mar	1 palmo Romano = 12 once
Sienna	Braccio	378.52	oncia	18.926	20	Сhа	
Palermo	Piede	227.84	oncia	18.987	12	Did	
Naples	Piede	232.35	oncia	19.363	12	Did	
Palermo	Palmo	242.05	oncia	20.171	12	Ca, Did, Kr, Cr	1 palmo = $12$ once
Palermo	Palmo	242.78	oncia	20.232	12	Dou	
Genoa	Palmo	248.08	oncia	20.674	12	Ca, Did, GDE, Mal, Mar, Kr, Cha	1 palmo = 12 once = 144 linee = 1728 punti
Rome	Palmo mercantile	248.99	oncia	20.749	12	Eu, Mar, Mal, Dou, Pa, Co	1 palmo = $12$ once
Sicily and							
Palermo	Palmo	257.80	oncia	21.483	12	Mal, Eu, Mar	1 palmo = 12 once = 144 linee = 1728 punti
Naples	Braccio	00.869	oncia	21.813	32	Ca	1 braccio = $2\%$ palmi = $32$ once

Matelica, Mondavio, Montetassiano, Montecosaro, Montefano, Montegiorgio, Montelupone, Montenovo, Morovalle, Offida, Orvieto, Osimo, 76 The Roman palmo and piede were used widely throughout parts of the Regions of Lazio and The Marche including: Ancona, Apiro, Ascoli Piceno, Belforte, Cagli, Camerino, Castelfidardo, Cingoli, Civitanova, Civitavecchia, Corinaldo, Fabriano, Fenigli, Fermo, Fossombrone, Frosinone, Loreto, Macerata, Pergola, Petriolo, Recanati, Rieti, Ripatransone, Roccacondrada, San Elpidio, San Ginesio, San Leo, Sanseverino, Sant'Agata Feltria, Sarnano, Sassoferato, Serra de' Conti, Serra San Quirico, Staffolo, Tolentino, Treia, Urbino, and Viterbo.

77 See footnote 76.

	Notes	1 canna = 8 palmi = 96 once = 480 minuti	1 $palmo = 12$ once = 60 minuti	1 canna = $8 palmi$	1 palmo = 12 once = 4 quarti = 60 minuti		1 piede = $12$ once		1 braccio = $12$ once	1 $pied = 12$ $pouce = 144$ lignes		1 piede = $12$ pollici = $144$ linee		1 piede = $12$ once	1 bavelle = 1 palmo	1 braccio = $20$ soldi = $240$ denari, etc.	1 braccio = $20$ soldi = $240$ denari, etc.				1 piede = $12$ once = $144$ linee	1 piede = $12$ once = $144$ linee		1 piede = $12$ once	1 piede = 12 once = 144 linee = 1440 decimi
	Source	Ca, Dou, Pa, Or, Cha	Dou, Cr	Ga, Mal, Mar	Did, Ga, Mal, Mar	Did	Kr	Ĉ	Kr	Mar, Cr		Mal, Mar	Did	Pa, Kr	Kr	Did	Co, Mar	Mar		Mar, Dou	EI, Dou, Mar, Ta2, Mal, Cr	Cr, Did, Dou, Pa, Or, Mal, Ta2, Mar	Dou	S	Did, Kr, Cr, Dou, Mal, Mar, Ta2
	Divi- sion	96	12	96	12	12	12	12	12	12		12	12	12	10	20	20	12		12	12	12	12	12	12
	Length in mm	21.834	21.835	21.973	21.973	21.994	22.558	22.575	22.653	26.000		26.340	26.882	26.919	27.341	27.409	27.560	27.659		27.858	28.374	28.575	28.650	28.762	28.978
	Sub- unit	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	pollice		pollice	oncia	oncia	oplos	oplos	oplos	oncia		oncia	oncia	oncia	oncia	oncia	oncia
	mm	2096.10	262.01	2109.36	263.67	263.93	270.70	270.90	271.83	312.00		316.08	322.58	323.03	273.41	548.17	551.20	331.91		334.30	340.49	342.90	343.80	345.14	347.74
contd)	Unit	Canna	Palmo	Canna	Palmo	Piede	Braccio	Piede	Piede	Pied	pu	Piede (Viennese Fuss)	Piede	Piede	Bavelle	Braccio di terra	Braccio a terra	Piede da fabbrica		Piede	Piede	Piede	Piede	Piede	Piede
(Table 11 contd)	Location	Naples	Naples	Naples	Naples	Genoa	Savoia	Verona	Rimini	Aosta	Trentino and	Trieste	Venice	Turin	Florence	Florence	Florence	Trento	Tirol and	Bolzano	Udine	Verona	Aquila	Udine	Venice

Notes	1 piede = $12$ once = $60$ minuti	1 piede = $12$ once	1 palmo = 12 once	1 piede da fabbrica = $12$ once		1 piede = $12$ once = $144$ linee	1 piede = $12$ once = $60$ minuti			1 piede = 12 once = 144 punti = 1728 atomi	_	1 $piede = 12$ once	1 piede da legname = $12$ once	1 piede = $12$ once	1 piede = $12$ once	1 piede = 12 once = 144 punti = 1728 atomi		1 braccio = 12 once = 144 punti = 1728 atomi	1 piede = $12$ once = $144$ punti = $1728$ atomi	1 piede = 12 once = 144 punti = 1728 atomi	1 piede = 12 once = 144 punti = 1728 atomi	1 piede = $12$ once	1 $piede = 12$ once	
Source	EI, Pa, Dou, Cr, Ta2, Mar, Mal	EI, Cr, Pa, Ta2, Mar, Mal, Pa, Kr	Did,Kr	Pa, Mal, Ce, Ta2, Mar	Kr, Did, Co	Co, Mar, Ta2	Mar, Pa	Mar, Dou	Did	Did, Kr, Co	Ca, Did, GDE, Eu, Mar, Ta1, Cr, Pa, Or, Mal, Dou	Pa, Cr, Mar	Pa, Mal, Ta2	$K_{\mathbf{r}}$	Ĉ	EI, Eu, Dou, Mar, Ta 1, Mal, Cr, Or, Pa	Did	Ca, Eu, Cr,Fr,Mal, Or,Pa,Dou,Mar	Dou, Fr, Or, Mar, Mal, Pa	Dou, Cr, Fr, Mar, Mal, Or	Dou, Pa, Cr, Fr, Mar, Or, Mal	Kr	Did, Kr, Or, Mal, Cr, Dou, Fr, Pa, Mar	Did Kr
Divi- sion	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
Length in mm	29.011	29.477	29.514	29.558	29.702	29.783	30.292	30.492	31.582	31.619	31.675	31.958	32.717	33.085	33.443	33.655	36.093	36.265	36.481	37.184	37.602	38.161	38.905	39.007
Sub- unit	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia
mm	348.14	353.72	354.17	354.70	356.42	357.39	363.50	365.90	378.98	379.43	380.10	383.50	392.60	397.03	401.31	403.85	433.12	435.19	437.77	446.20	451.22	457.93	466.86	468.08
Unit	Piede da fabbrica	Piede da fabbrica	Palmo	Piede da fabbrica	Piede	Piede	Piede	Piede	Piede or Braccio	Piede	Piede agrimensorio	Piede	Piede da legname	Piede	Piede	Piede	Piede	Piede	Piede	Piede	Piede	Piede	Piede	Braccio
Location	Pesaro	Urbino	Padua	Senigallia	Vicenza	Padua	Perugia	Trento	Bologna	Bologna	Bologna	Urbino	Senigallia	Milan	Ferrara	Ferrara	Turin	Milan	Bergamo	Sondrio	Como	Pavia	Mantua	Brescia

(Tabbe 11 contd)

	Notes		1 piede = 12 once = 144 punti = 1728 atomi	1 piede = $12$ once	1 piede = $12$ once = $144$ punti = $1728$ atomi		1 piede = $12$ once = $144$ punti = $1728$ atomi		1 piede = 12 once = 144 punti = 1728 atomi	1 piede = $2 palmi = 12$ once			1 piede = $12$ once = $144$ punti = $1728$ atomi	1 piede legale $= 8$ once	1 piede = 12 once = 144 punti = 1728 atomi	1 piede = $2$ palmi = $12$ once = $144$ punti	1 piede = $10$ once = $100$ punti	1 piede = $12$ once = $144$ punti	1 piede = 12 once = 144 punti = 1728 atomi				1 piede = $12$ once	1 piede = $10$ once	1 piede = $10$ once	$1 \ piede = 10 \ once = 100 \ punti$
	Source	Did	Fr, Mal, Cr, Dou, Mar	Cr, Or, Mal, Dou	EI, Dou, Cr, Fr, Mar, Or, Mal, Pa	Did	Fr, Mar, Dou, Mal, Ta 1, Mal, Pa	Did	Dou, Pa, Or, Mal, Fr, Mar	Kr	Dou	Дои	Ca, Or, Mal, Eu, Cr, Mar, Dou	Cr, Dou, Eu, Mar, Mal	Pa, Eu, Ta1, Cr, Dou, Mal, Mar, Or	Mar, GDE, Eu, Ca, Dou	Cr, Dou, Or, Mal, Mar, Ta 1, Pa	Ta1, Or, Cr, Dou, Mar, Pa, Or, Mal	Did, Kr, Mar, Mal, Pa, Or, Dou, Eu, Ca	Did,Kr	Did	Dou, Pa, Cr, Mal	Dou, Pa, Cr, Mal	Dou, Mar, Or, Mal, Ta1, Pa, Cr	Cr, Dou, Pa, Or, Mal, Mar, Ta 1	Pa, Or, Mal, Mar, Cr
	Divi- sion	12	12	12	12	12	12	12	12	9	12	<b>∞</b>	12	∞	12	12	10	12	12	12	12	12	12	10	10	10
	Length in mm	39.101	39.130	39.249	39.330	39.352	39.622	39.759	40.295	41.845	42.814	42.814	42.814	42.814	43.587	43.725	43.966	44.242	45.431	45.549	45.868	46.542	46.542	47.977	48.821	58.461
	Sub- unit	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia
	шш	469.21	469.57	470.99	471.95	472.22	475.47	477.11	483.54	251.07	513.77	342.51	513.77	342.51	523.05	524.70	439.66	530.90	545.17	546.59	550.42	558.51	558.50	479.77	488.21	584.61
ontd)	Unit	Piede	Piede	Piede	Piede agrimensorio	Piede	Piede	Piede	Piede	Palmo	Piede liprando	Piede manuale	Piede (piede liprando)	Piede legale	Piede	Piede	Piede agrimensorio	Piede	Braccio o piede	Braccio	Piede	Piede agrimensorio	Piede	Piede	Piede	Piede agrimensorio
(Table 11 contd)	Location	Mantua	Piacenza	Brescia	Pavia	Brescia	Brescia	Brescia	Cremona	Sardinia	Genoa	Genoa	Turin	Turin	Modena	Cagliari	Imola	Reggio	Parma	Piacenza	Parma	Macerata	Senigallia	Faenza	Forlì	Ravenna

Table 12. Table the oncia, soldo or pollice in the Italian peninsula arranged according to location

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Location	Unit	mm	Sub-	Length Divi-	Divi-	Source	Notes
			unit	in mm	sion		
Aosta	Pied	312.00	pollice	26.000	12	Mar, Cr	1 pied = 12 pouce = 144 lignes
Aquila	Piede	343.80	oncia	28.650	12	Dou	
Bergamo	Piede	437.77	oncia	36.481	12	Dou, Fr, Or, Mar, Mal, Pa	1 piede = 12 once = 144 punti = 1728 atomi
Bologna	Piede or Braccio	378.98	oncia	31.582	12	Did	
Bologna	Piede	379.43	oncia	31.619	12	Did, Kr, Co	1 piede = 12 once = 144 punti = 1728 atomi
Bologna	Piede agrimensorio	380.10	oncia	31.675	12	Ca, Did, GDE, Eu, Mar, Ta1, Cr, Pa, Or, Mal, Dou	1 piede = $12$ once = $144$ punti = $1728$ atomi
Brescia	Braccio	468.08	oncia	39.007	12	Did,Kr	
Brescia	Piede	470.99	oncia	39.249	12	Cr, Or, Mal, Dou	1 piede = $12$ once
Brescia	Piede	472.22	oncia	39.352	12	Did	
Brescia	Piede	475.47	oncia	39.622	12	Fr, Mar, Dou, Mal, Ta 1, Mal, Pa	1 piede = 12 once = 144 punti = 1728 atomi
Brescia	Piede	477.11	oncia	39.759	12	Did	
Cagliari	Piede	524.70	oncia	43.725	12	Mar, GDE, Eu, Ca, Dou	1 piede = $2 \text{ palmi} = 12 \text{ once} = 144 \text{ punti}$
Como	Piede	451.22	oncia	37.602	12	Dou, Pa, Cr, Fr, Mar, Or, Mal	1 piede = 12 once = 144 punti = 1728 atomi
Cremona	Piede	483.54	oncia	40.295	12	Dou, Pa, Or, Mal, Fr, Mar	1 piede = 12 once = 144 punti = 1728 atomi
Faenza	Piede	479.77	oncia	47.977	10	Dou, Mar, Or, Mal, Ta 1, Pa, Cr	1 piede = $10$ once
Ferrara	Piede	401.31	oncia	33.443	12	S	1 piede = $12$ once
Ferrara	Piede	403.85	oncia	33.655	12	EI, Eu, Dou, Mar, Ta1, Mal, Cr, Or, Pa	1 piede = 12 once = 144 punti = 1728 atomi
Florence	Bavelle	273.41	oplos	27.341	10	Kr	1 bavelle = 1 $palmo$
Florence	Braccio di terra	548.17	oplos	27.409	20	Did	1 braccio = $20$ soldi = $240$ denari, etc.
Florence	Braccio a terra	551.20	oplos	27.560	20	Co,Mar	1 braccio = $20$ soldi = $240$ denari, etc.
Forlì	Piede	488.21	oncia	48.821	10	Cr,Dou,Pa,Or,Mal,Mar,Ta1	1 piede = $10$ once

(Table 12 contd)	ontd)						
Location	Unit	шш	Sub- unit	Length in mm	Divi- sion	Source	Notes
Genoa	Palmo	248.08	oncia	20.674	12	Ca, Did, GDE, Mal, Mar, Kr, Cha	1 palmo = 12 once = 144 linee = 1728 punti
Genoa	Piede	263.93	oncia	21.994	12	Did	
Genoa	Piede liprando	513.77	oncia	42.814	12	Dou	
Genoa	Piede manuale	342.51	oncia	42.814	œ	Dou	
Imola	Piede agrimensorio	439.66	oncia	43.966	10	Cr, Dou, Or, Mal, Mar, Ta1, Pa	1 piede = $10$ once = $100$ punti
Macerata	Piede agrimensorio	558.51	oncia	46.542	12	Dou, Pa, Cr, Mal	
Mantua	Piede	466.86	oncia	38.905	12	Did, Kr, Or, Mal, Cr, Dou, Fr, Pa, Mar	1 piede = $12$ once
Mantua	Piede	469.21	oncia	39.101	12	Did	
Milan	Piede	397.03	oncia	33.085	12	Kr	1 piede = $12$ once
Milan	Piede	435.19	oncia	36.265	12	Ca, Eu, Cr, Fr, Mal, Or, Pa, Dou, Mar	1 braccio = 12 once = 144 punti = 1728 atomi
Modena	Piede	523.05	oncia	43.587	12	Pa, Eu, Ta1, Cr, Dou, Mal, Mar, Or	1 piede = 12 once = 144 punti = 1728 atomi
Naples	Piede	232.35	oncia	19.363	12	Did	
Naples	Braccio	00.869	oncia	21.813	32	Ca	1 braccio = $2 \%$ palmi = $32$ once
Naples	Canna	2096.10	oncia	21.834	96	Ca, Dou, Pa, Or, Cha	1 canna = 8 palmi = 96 once = 480 minuti
Naples	Palmo	262.01	oncia	21.835	12	Dou, Cr	1 $palmo = 12$ once = 60 minuti
Naples	Canna	2109.36	oncia	21.973	96	Ga, Mal, Mar	$1 \ canna = 8 \ palmi$
Naples	Palmo	263.67	oncia	21.973	12	Did, Ga, Mal, Mar	1 palmo = 12 once = 4 quarti = 60 minuti
Padua	Palmo	354.17	oncia	29.514	12	Did,Kr	1 palmo = 12 once
Padua	Piede	357.39	oncia	29.783	12	Co, Mar, Ta2	1 piede = $12$ once = $144$ linee
Palermo	Piede	227.84	oncia	18.987	12	Did	
Palermo	Palmo	242.05	oncia	20.171	12	Ca, $Did$ , $Kr$ , $Cr$	1 palmo = 12 once
Palermo	Palmo	242.78	oncia	20.232	12	Дон	
Parma	Braccio o piede	545.17	oncia	45.431	12	Did, Kr, Mar, Mal, Pa, Or, Dou, Eu, Ca	1 piede = 12 once = 144 punti = 1728 atomi
Parma	Piede	550.42	oncia	45.868	12	Did	

Notes		1 piede = $12$ once	1 piede = 12 once = 144 punti = 1728 atomi	1 piede = $12$ once = $60$ minuti	1 piede = $12$ once = $60$ minuti	1 piede = 12 once = 144 punti = 1728 atomi		1 piede = $10$ once = $100$ punti	1 piede = $12$ once = $144$ punti	1 braccio = $12$ once	Dou ays that this piede was still in use in 1840.	1 piede romano = $16$ once	1 palmo = 12 once = 120 decimi = 60 minuti	1 piede = $1 \text{ 1/3 palmi}$ = $16 \text{ once}$	1 palmo = 12 once	1 $piede = 2 palmi = 12 once$	1 piede = $12$ once	1 piede da fabbrica = $12$ once	1 piede da legname = $12$ once	1 piede = $12$ once		1 palmo = 12 once = 144 linee = 1728 punti		1 piede = 12 once = 144 punti = $1728$ atomi
Source		$K_{\mathbf{r}}$	EI, Dou, Cr, Fr, Mar, Or, Mal, Pa	Mar, Pa	EI, Pa, Dou, Cr, Ta2, Mar, Mal	Fr, Mal, Cr, Dou, Mar	Did, Kr	Pa, Or, Mal, Mar, Cr	Ta1, Or, Cr, Dou, Mar, Pa, Or, Mal	Kr	Did, Kr, Dou	Ca, Eu	Cha, Did, Co, Or, Mal, Mar, Dou, Eu	Dou, Cr, Mal, Mar, Cr, Pa, Kr	Eu, Mar, Mal, Dou, Pa, Co	Kr	K	Pa,Mal, Ce, Ta2,Mar	Pa, Mal, Ta2	Dou, Pa, Cr, Mal		Mal, Eu, Mar	Cha	Dou, Cr, Fr, Mar, Mal, Or
Divi-	sion	12	12	12	12	12	12	10	12	12	16	16	12	16	12	9	12	12	12	12		12	20	12
Length Divi-	ın mm	38.161	39.330	30.292	29.011	39.130	45.549	58.461	44.242	22.653	18.406	18.563	18.619	18.619	20.749	41.845	22.558	29.558	32.717	46.542		21.483	18.926	37.184
Sub-	unıt	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia	oncia		oncia	oncia	oncia
mm		457.93	471.95	363.50	348.14	469.57	546.59	584.61	530.90	271.83	294.50	297.00	223.42	297.90	248.99	251.07	270.70	354.70	392.60	558.50		257.80	378.52	446.20
Unit		Piede	Piede agrimensorio	Piede	Piede da fabbrica	Piede	Braccio	Piede agrimensorio	Piede	Piede	Piede antico	Piede romano	Palmo (architettonico)	Piede	Palmo mercantile	Palmo	Braccio	Piede da fabbrica	Piede da legname	Piede		Palmo	Braccio	Piede
Location		Pavia	Pavia	Perugia	Pesaro	Piacenza	Piacenza	Ravenna	Reggio	Rimini	Rome	Rome	Rome	Rome	Rome	Sardinia	Savoia	Senigallia	Senigallia	Senigallia	Sicily and	Palermo	Sienna	Sondrio

Location         Unit         mm         Sub- Longth Division         Source         Notes           The Marche**         Piede         335.10         unit         in mm         sion         C,EI,Dou,Pa,Ta2,Mal         1 piede da leguane = 18 once           The Marche**         Piede         335.10         oncia         18 617         18         C,EI,Dou,Pa,Ta2,Mal         1 piede da leguane = 18 once           Tironla and Bolzano         Piede         334.30         oncia         27.42         12         Mar,Dou         1 piede e = 12 pollici = 144 linee           Tironla and Bolzano         Piede da fabritia         331.91         oncia         27.659         12         Mar,Dou         1 piede = 12 pollici = 144 linee           Tironla and Piede Virennese Fluss)         316.08         pollici         26.59         12         Mar,Dou         1 piede = 12 pollici = 144 linee           Turin         Piede da fabritia         33.03         oncia         26.91         12         Mar,Dou         1 piede = 12 pollici = 144 linee           Turin         Piede piede liprando         513.77         oncia         26.93         12         A.O.O.Mal,Ba,A.C.Mar,Dou         1 piede = 12 pollici = 144 linee           Turin         Piede pigate         345.14         oncia         28.34	(rapic 17 cours)	mar)						
rcche <sup>78</sup> Piede 335.10 oncia 18.617 18 Cr,El,Dou,Pa,Ta2,Mal rcche <sup>78</sup> Palmo 223.42 oncia 18.619 12 Mar dar dar blede 223.42 oncia 18.619 12 Mar and a piede Agbbrica 334.30 oncia 25.340 12 Mar,Dou blede Agbbrica 331.91 oncia 25.340 12 Mar,Dou blede Agbbrica 330.3 oncia 25.90 12 Mar,Dou blede Gilgede liprando) 513.77 oncia 26.919 12 Packe Chou,Bu,Kr Piede Lagale 343.12 oncia 36.093 12 Did Piede Liprando) 513.77 oncia 42.814 12 Ca,Or,Mal,Eu,Cr,Mar,Mal Piede Bgale 345.14 oncia 28.374 12 El,Cou,Mar,Ta2,Mar,Mal Piede Agbbrica 353.72 oncia 28.374 12 El,Cr,Pa,Ta2,Mar,Mal Piede 353.72 oncia 28.375 12 Ca,Or,Mar,Mar,Mar,Biede 353.72 oncia 28.375 12 Ca,Or,Mar,Mar,Mar,Mar,Mar,Mar,Mar,Mar,Mar,Ma	Location	Unit	шш	Sub- unit	Length in mm	Divi- sion	Source	Notes
rche <sup>79</sup> Palmo 223.42 oncia 18.619 12 Mar  d  d  A Piede 334.30 oncia 27.858 12 Mar,Dou  Piede da fabbrica 31.91 oncia 27.659 12 Mar,Dou  Piede da fabbrica 33.03 oncia 26.340 12 Mar,Dou  Piede liprando 323.03 oncia 26.319 12 Mar,Dou  Piede liprando 313.71 oncia 36.093 12 Did  Piede legale 343.12 oncia 36.093 12 Did  Piede legale 343.12 oncia 28.314 12 Ca,Or,Mal,Eu,Cr,Mar,Mal  Piede legale 345.14 oncia 28.762 12 Ca,Or,Mal,Eu,Cr,Mar,Mal  Piede a fabbrica 335.72 oncia 28.762 12 Ca,Or,Mal,Bu,Cr  Piede a fabbrica 335.72 oncia 28.762 12 Ca,Or,Mar,Mal  Piede a fabbrica 335.72 oncia 28.762 12 Ca,Or,Mar,Mal  Piede a fabbrica 335.72 oncia 28.762 12 Ca,Or,Mar,Mal  Piede a fabbrica 345.14 oncia 28.762 12 Ca,Or,Mar,Mal  Piede 332.58 oncia 28.762 12 Ca,Or,Mar,Mal  Piede a fabbrica 345.74 oncia 28.872 12 Oid, Eu,Cr,Mar  Piede 347.74 oncia 28.872 12 Ca,Or,Mal,Mar,Ta2  Piede 342.90 oncia 28.575 12 Ca,Did,Dou,Mal,Mar,Ta2  Piede 342.90 oncia 28.575 12 Cr,Did,Dou,Mal,Mar,Ta2  Piede 342.90 oncia 28.575 12 Cr,Did,Dou,Mal,Ta2,Mar  Piede St.Cr, Mar,Did,Co	The Marche		335.10	oncia	18.617	18	Cr, EI, Dou, Pa, Ta2, Mal	1 piede da legname = $18$ once
Piede (Viennese Fuss)   316.08   pollice   27.858   12   Mar, Dou	The Marche		223.42	oncia	18.619	12	Mar	1 palmo Romano = 12 once
Piede (Viennese Fuss)         316.08         pollice         26.340         12         Mal,Mar           Piede da fabbria         331.91         oncia         27.659         12         Mal,Mar           Piede         365.90         oncia         27.659         12         Mat,Dou           Piede         323.03         oncia         26.919         12         Mat,Dou           Piede (piede liprando)         513.77         oncia         42.814         12         Ca,Or,Mal,Eu,Cr,Mar,Dou           Piede liprando)         513.77         oncia         42.814         8         Cr,Dou,Bu,Mar,Ta2,Mal           Piede liprando)         513.77         oncia         28.374         12         Ca,Or,Mal,Eu,Cr,Mar,Dou           Piede liprando)         345.14         oncia         28.762         12         Cr,Dou,Mar,Ta2,Mal,Cr           Piede da fabbrica         353.72         oncia         28.762         12         El,Cr,Pa,Ta2,Mar,Mal,Pa,Kr           Piede         333.50         oncia         28.878         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         347.74         oncia         28.978         12         Cr,Did,Dou,Mal,Ta2,Mar           Piede         270.90         oncia         28.575	Tirol and	Diede	334 30	.000	27.858	12	Mar Dou	
Piede (Viennese Fuss)         316.08         pollice         26.340         12         Mal, Mar           Piede da fabbrica         331.91         oncia         27.659         12         Mar           Piede         365.90         oncia         30.492         12         Mar, Dou           Piede         433.12         oncia         26.919         12         Pa, Kr           Piede liprando         513.77         oncia         42.814         12         Ca,Or,Mal,Eu,Cr,Mar,Dou           Piede lighed liprando         340.49         oncia         28.374         12         Ca,Or,Mal,Eu,Cr,Mar,Dou           Piede legale         340.49         oncia         28.374         12         Ca,Or,Mar,Ta2,Mar,Aal           Piede         345.14         oncia         28.762         12         Co,Dou,Mar,Mar,Aal           Piede         353.72         oncia         29.477         12         EI,Cr,Pa,Ta2,Mar,Mal,Pa,Kr           Piede         325.58         oncia         26.882         12         Did           Piede         347.74         oncia         28.978         12         Co           Piede         342.90         oncia         22.575         12         Cr,Did,Dou,Mal,Mar,Ta2 <tr< td=""><td>Trentino and</td><td></td><td></td><td></td><td></td><td>1</td><td>uo Timit</td><td></td></tr<>	Trentino and					1	uo Timit	
Piede da fibbhria         331.91         oncia         27.659         12         Mar           Piede         365.90         oncia         30.492         12         Mar, Dou           Piede         323.03         oncia         26.919         12         Pa, Kr           Piede liprando         513.77         oncia         42.814         12         Ca, Or, Mal, Eu, Cr, Mar, Dou           Piede legale         342.51         oncia         42.814         8         Cr, Dou, Eu, Mar, Tad           Piede         340.49         oncia         28.374         12         El, Dou, Mar, Tad, Mar, Tad           Piede         345.14         oncia         28.762         12         Ca, Or, Mar, Mar, Mar, Mar, Mar, Mar, Mar, Ma		Piede (Viennese Fuss)	316.08	pollice	26.340	12	Mal, Mar	1 piede = $12$ pollici = $144$ linee
Piede         365.90         ondia         30.492         12         Mar, Dou           Piede         323.03         ondia         26.919         12         Pa, Kr           Piede liprando         513.77         ondia         42.814         12         Ca, Or, Mal, Eu, Cr, Mar, Dou           Piede legale         342.51         ondia         42.814         8         Cr, Dou, Eu, Mar, Mal           Piede         340.49         ondia         28.374         12         El, Dou, Mar, Ta2, Mar, Cr           Piede         345.14         ondia         28.762         12         Ca, Or, Mar, Ta2, Mar, Mal, Pa, Kr           Piede         353.72         ondia         29.477         12         El, Cr, Pa, Ta2, Mar, Mal, Pa, Kr           Piede         383.50         ondia         28.82         12         Did           Piede         347.74         ondia         28.882         12         Did           Piede         270.90         ondia         22.575         12         Co           Piede         342.90         ondia         28.575         12         Cr, Did, Dou, Mal, Ta2, Mar           Piede         342.90         ondia         28.575         12         Cr, Did, Dou, Pa, Or, Mal, Ta2, Mar	Trento	Piede da fabbrica	331.91	oncia	27.659	12	Mar	
Piede         323.03         ondia         26.919         12         Pa,Kr           Piede (piede liprando)         513.77         ondia         36.093         12         Did           Piede legale         342.51         ondia         42.814         12         Ca,Or,Mal,Eu,Cr,Mar,Dou           Piede         340.49         ondia         28.374         12         El,Dou,Eu,Mar,Mal           Piede         345.14         ondia         28.762         12         Cr,Dou,Eu,Mar,Mal           Piede         3453.72         ondia         28.762         12         El,Cr,Mar           Piede         383.50         ondia         29.58         12         Pa,Cr,Mar           Piede         342.54         ondia         28.882         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         347.74         ondia         28.978         12         Did,Kr,Cr,Dou,Mal,Ta2,Mar           Piede         270.90         ondia         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         ondia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         ondia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar	Trento	Piede	365.90	oncia	30.492	12	Mar, Dou	
Piede         433.12         ondia         36.093         12         Did           Piede legale         342.51         ondia         42.814         12         Ca,Or,Mal,Eu,Cr,Mar,Dou           Piede legale         342.51         ondia         42.814         8         Cr,Dou,Eu,Mar,Mal           Piede         340.49         ondia         28.374         12         El,Dou,Mar,Ta2,Mar,Mal           Piede         345.14         ondia         28.762         12         El,Cr,Pa,Ta2,Mar,Mal,Pa,Kr           Piede         383.50         ondia         29.78         12         Pa,Cr,Mar           Piede         325.58         ondia         26.882         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         270.90         ondia         28.978         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         ondia         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         ondia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar	Turin	Piede	323.03	oncia	26.919	12	Pa, Kr	1 piede = $12$ once
Piede (piede liprando)         513.77         ondá         42.814         12         Ca,Or,Mal, Eu, Cr,Mar, Dou           Piede legale         342.51         ondá         42.814         8         Cr,Dou, Eu, Mar, Mal           Piede         340.49         ondá         28.374         12         El,Dou,Mar, Ta2,Mar,Mal           Piede         345.14         ondá         28.762         12         Co           Piede da fabbrica         383.50         ondá         29.477         12         El,Cr,Par,Ta2,Mar,Mal,Pa,Kr           Piede         322.58         ondá         26.882         12         Did           Piede         347.74         ondá         28.978         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         270.90         ondá         22.575         12         Co           Piede         342.90         ondá         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         ondá         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         ondá         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar	Turin	Piede	433.12	oncia	36.093	12	Did	
Piede legale         342.51         onda         42.814         8         Cr,Dou,Eu,Mar,Mal           Piede         340.49         onda         28.374         12         El,Dou,Mar,Ta2,Mal,Cr           Piede         345.14         onda         28.762         12         Co           Piede         383.72         onda         29.477         12         El,Cr,Par,Ta2,Mar,Mal,Pa,Kr           Piede         322.58         onda         26.882         12         Did           Piede         347.74         onda         28.978         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         270.90         onda         22.575         12         Co           Piede         342.90         onda         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         onda         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         onda         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar	Turin	Piede (piede liprando)	513.77	oncia	42.814	12	Ca, Or, Mal, Eu, Cr, Mar, Dou	1 piede = $12$ once = $144$ punti = $1728$ atomi
Piede         340.49         onda         28.374         12         El,Dou,Mar,Ta2,Mal,Cr           Piede da fabbrica         345.14         onda         28.762         12         Co           Piede da fabbrica         353.72         onda         29.477         12         El,Cr,Pa,Ta2,Mar,Mal,Pa,Kr           Piede         383.50         onda         28.82         12         Pa,Cr,Mar           Piede         347.74         onda         28.978         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         270.90         onda         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         onda         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         onda         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar	Turin	Piede legale	342.51	oncia	42.814	<b>∞</b>	Cr,Dou,Eu,Mar,Mal	1 piede legale $= 8$ once
Piede         345.14         onda         28.762         12         Co           Piede da fabbrica         353.72         oncia         29.477         12         El,Cr,Pa, Ta2,Mar,Mal,Pa,Kr           Piede         383.50         oncia         31.958         12         Pa,Cr,Mar           Piede         322.58         oncia         28.978         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         270.90         oncia         22.575         12         Co           Piede         342.90         oncia         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         356.42         oncia         29.702         12         Kr,Did,Co	Udine	Piede	340.49	oncia	28.374	12	EI, Dou, Mar, Ta2, Mal, Cr	1 piede = $12$ once = $144$ linee
Piede da fabbrica         353.72         oncia         29.477         12         El,Cr,Pa,Ta2,Mar,Mal,Pa,Kr           Piede         383.50         oncia         26.882         12         Pa,Cr,Mar           Piede         347.74         oncia         28.978         12         Did           Piede         270.90         oncia         28.978         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         270.90         oncia         28.575         12         Co,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         oncia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         356.42         oncia         29.702         12         Kr,Did,Co	Udine	Piede	345.14	oncia	28.762	12	Ĉ	1 piede = $12$ once
Piede         383.50         onaia         31.958         12         Pa,Cr,Mar           Piede         322.58         onaia         26.882         12         Did           Piede         347.74         onaia         28.978         12         Did,Kr,Cr,Dou,Mal,Mar,Ta2           Piede         270.90         onaia         22.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         342.90         onaia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         356.42         onaia         29.702         12         Kr,Did,Co	Urbino	Piede da fabbrica	353.72	oncia	29.477	12	EI, Cr, Pa, Ta2, Mar, Mal, Pa, Kr	1 piede = $12$ once
Piede         322.58         ontia         26.882         12         Did           Piede         347.74         ontia         28.978         12         Did,Kt,Ct,Dou,Mal,Mar,Ta2           Piede         270.90         ontia         22.575         12         Co           Piede         342.90         ontia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         356.42         ontia         29.702         12         Kr,Did,Co	Urbino	Piede	383.50	oncia	31.958	12	Pa, Cr, Mar	1 piede = $12$ once
Piede         347.74         oncia         28.978         12         Did,Kr,Cr,Dou,Mal,Mar,Tå2           Piede         270.90         oncia         22.575         12         Co           Piede         342.90         oncia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           Piede         356.42         oncia         29.702         12         Kr,Did,Co	Venice	Piede	322.58	oncia	26.882	12	Did	
Piede         270.90         oncia         22.575         12         Co           Piede         342.90         oncia         28.575         12         Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar           1         Piede         356.42         oncia         29.702         12         Kr,Did,Co	Venice	Piede	347.74	oncia	28.978	12	Did, Kr, Cr, Dou, Mal, Mar, Ta2	1 piede = $12$ once = $144$ linee = $1440$ decimi
Piede 342.90 oncia 28.575 12 Cr,Did,Dou,Pa,Or,Mal,Ta2,Mar 1 Piede 356.42 oncia 29.702 12 Kr,Did,Co	Verona	Piede	270.90	oncia	22.575	12	Ŝ	
Piede 356.42 oncia 29.702 12	Verona	Piede	342.90	oncia	28.575	12	Cr, Did, Dou, Pa, Or, Mal, Ta2, Mar	1 piede = $12$ once = $144$ linee
	Vicenza	Piede	356.42	oncia	29.702	12	$K_{\mathbf{f}}$ , $Did$ , $Co$	

<sup>&</sup>lt;sup>78</sup> See footnote 76.